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A THREE LAYER FLUID MODEL FOR ESTIMATING OCEAN BOTTOM REFLECTIV--ETC(U)

JAN 63 B F COLE

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U. S. NAVY UNDERWATER SOUND LABORATORY
FORT TRUMBULL, NEW LONDON, CONNECTICUT

6

A THREE LAYER FLUID MODEL FOR ESTIMATING OCEAN BOTTOM REFLECTIVITY AT VARIOUS FREQUENCIES,

-2

By

10 Bernard F. Cole

MOST Project

14 USL-TM
USL Technical Memorandum No. 905-4-63

11 8 January 1963

9 Technical memo,

12 30p.

INTRODUCTION

The adoption of the bottom bounce mode of underwater sound propagation for sonar applications has made a consideration of ocean bottom sound reflectivity essential. The following discussion is an attempt to: (1) Relate observed ocean bottom physical and acoustic properties, and (2) present a better theoretical approach for comparison with existing measurements than those previously employed.

OCEAN BOTTOM PHYSICAL AND ACOUSTIC PROPERTIES

A physical analysis of shallow water sediments from the Pacific Ocean by Hamilton et al., (1956)[1] showed a correlation between the velocity and the density, between the velocity and the porosity, and between the velocity and the median grain size of the sediment layer. A general increase in velocity with increasing median grain size and density and decreasing porosity was noted. It is interesting and

[1]Hamilton, E., Shumway, G., Menard, H., Shipek, C., "Acoustic and Other Physical Properties of Shallow Water Sediments," Journal of the Acoustical Society of America, 28-1, 1 (January 1956).

perhaps significant that the same correlation was observed in a physical analysis of Atlantic deep sea sediment cores by Sutton, et al., (1957)[2]. Although both studies showed a strong positive correlation to exist between the median grain size and the velocity, Sutton interpreted the median grain size as a factor directly affecting the velocity, while Hamilton explained the correlation as a result of the velocity-porosity relationship, and a strong negative correlation between the median grain size and the porosity.

Sutton found the compressional wave velocity in the ocean bottom unconsolidated sediments studied to be well represented by:

$$v' = 2.093 - (0.0414 \pm 0.0060)\,\phi + (0.00135 \pm 0.00038)\,\gamma - (0.44 \pm 0.15)\,\eta$$

where

v' = compressional wave velocity in km/sec.

ϕ = median grain size in phi units

γ = % of carbonate content[3]

η = porosity

Later sediment sound speed and absorption measurements by Shumway (1960)[4] found the velocity to be related to the porosity, the rigidity, the pressure, the temperature, and the compressibility of the grain aggregate, and correlated the absorption with the porosity and the median grain size. Shumway also found the absorption to vary with the frequency raised to the 1.79 ± 0.98[5] power.

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[2]Sutton, G., Berckhemer, H., Nafe, J., "Physical Analysis of Deep Sea Sediments," Geophysics, 24-4, 779 (October 1957).

[3]Sutton also found a positive correlation between the carbonate content and the velocity.

[4]Shumway, G., "Sound Speed and Absorption Studies of Marine Sediments by a Resonance Method" - Parts I and II, Geophysics, 25, 451 and 659 (1960).

[5]0.98 is the standard deviation.

The variation of sediment velocity, porosity and density with depth in the sediment has been considered by both Hamilton (1959)[6] and Nafe and Drake (1957)[7]. Nafe and Drake (1961)[8] have also written an excellent summary of various studies involving sediment physical properties, which include all the previously mentioned studies.

In an attempt to find some correlation between sonar bottom loss mesurements and bottom composition, sediment cores from the Atlantic were compared with bottom loss measurements according to geographic position. The results obtained from this comparison indicated that areas containing sand layers generally provide very good reflectivity, while areas containing only silt or clay layers provide average to poor reflectivity.[9] These observations are consistent with those of other studies: Liebermann (1948)[10] designated mud as the poorest reflector; Ewing et al., (1960)[11] found indications that silt is a poor reflector; Moore (1960)[12]

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[6]Hamilton, E. L., "Thickness and Consolidation of Deep Sea Sediments", Bulletin of the Geological Society of America, 70-11, 1399 (November 1959).

[7]Nafe, J. E., Drake, C. L., "Variation with Depth in Shallow and Deep Water Marine Sediments of Porosity, Density, and the Velocities of Compressional and Shear Waves", Geophysics, 22-3, 523 (July 1957).

[8]Nafe, J. E., Drake, C. L., "Physical Properties of Marine Sediments", Lamont Geological Observatory Technical Report No. 2 (June 1961).

[9]Remarks of Mr. T. G. Bell from "Proceedings of Conference on Sound Reflection from the Sea Bottom", AVCO Marine Electronics Dept./ Lamont Geological Observatory Report #MED-62-1019, p 24 (January 1962).

[10]Liebermann, L. N., "Reflection of Sound from Coastal Sea Bottoms", Journal of the Acoustical Society of America, 20, 35 (1948).

[11]Ewing, J., Luskin, B., Roberts, A., and Hirshman, J., "Sub Bottom Reflection Measurements on the Continental Shelf, Bermuda Banks, West Indies Arc, and in the West Atlantic Basins", Journal of Geophysical Research, 65-9, 2849 (September 1960).

[12]Moore, D. G., "Acoustic Reflection Studies of the Continental Shelf and Slope Off Southern California", Bulletin of the Geological Society of America, 71-8, 1121 (1960).

observed that clean sand is the hardest to penetrate acoustically, while silty clays are most easily penetrated; Smith (1960)[13] noticed reflection from sand, but penetration of silt and clay; Mackenzie (1960)[14] indicated that sand is a good reflector; Davidian (1961)[15] showed high reflection losses over a silt bottom; Hasse (1961)[16] concluded that bottom loss for sand bottoms in shallow water is appreciably less than that for mud bottoms, and McClure et al., (1958)[17] experienced good reflection from sand layers at 3.8 kc.

From the above discussed correlation between the median grain size and the velocity, and between the median grain size and the reflectivity, we would expect also a correlation between the sediment reflectivity and the sediment velocity. Such a correlation has been observed. Fry (1960)[18] and Fry and Raitt (1961)[19] have found that low velocity sediments provide poor reflectivity, while Katz and Ewing (1960)[20], by obtaining good sub-bottom reflections in certain areas, have indicated good penetration of such layers. A more complete discussion of this low-velocity layer is furnished by Ewing and Nafe (1961)[21].

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[13] Smith, W. O., "Recent Underwater Surveys Using Low Frequency Sound to Locate Shallow Bedrock", Bulletin of the Geological Society of America, 69-1 (1958).

[14] Mackenzie, K.V., "Reflection of Sound from Coastal Bottoms", Journal of the Acoustical Society of America, 32-2, 221 (February 1960).

[15] Davidian, M., "Sounding Intensity Study Off Green Cay in Tongue of the Ocean", Report of NRL Progress, (November 1961).

[16] Hasse, R.,Jr., "COLOSSUS II Summary Report", USL Report No. 513 (July 1961) (Confidential).

[17] McClure, C. D., Nelson, H. F., Huckabay, W. B., "Marine Sonoprobe System, New Tool for Geologic Mapping", Bulletin of American Association of Petroleum Geologist, 701 (1958).

[18] Fry, J. C., "The Reflection of Sound from the Deep Sea Floor", USAG Journal, 11-2 (April 1961).

[19] Fry, J. C., Raitt, R. W., "Sound Velocities at the Surface of Deep Sea Sediments", Journal of Geophysical Research, 66-2, 589 (February 1961)

[20] Katz, S., Ewing, M., "Seismic-Refraction Measurements in the Atlantic Ocean Part VII Atlantic Ocean Basin, West of Bermuda", Bulletin of the Geological Society of America, 67-4, 475 (1960).

[21] Ewing, J., Nafe, J., "The Unconsolidated Sediments", Lamont Geological Observatory Technical Report No. 3 (June 1961).

THEORETICAL BOTTOM REFLECTIVITY

Previous investigations involving ocean bottom reflectivity have included theoretical analysis as well as experimental observations. Fry[22] compared his calculated sediment reflection coefficients with the Rayleigh reflection coefficients for different sediments, and Hamilton[23] used the Mackenzie form of the Rayleigh equation to calculate the reflection loss for a number of sediments. Mackenzie (1960)[24] later presented a modified Rayleigh expression, which R. W. Morse had derived, to consider attenuation in the bottom fluid. This modified Rayleigh expression also appeared in USL Report No. 255 (1955)[25] in a different form, and was extended to consider two fluid layers, the second again containing attenuation, over a rigid bottom layer.

The USL Report No. 255 extension of the Rayleigh expression was rederived, as an initial effort of the work reported in the present paper, to clarify the form of the equation and was programmed in Fortran.[26] From the resultant calculations, it was evident that a further extension to a model consisting of three fluid layers of finite impedence would be useful.

The expression for the reflection amplitude coefficient was derived for the three-layer model[27], assuming absorption and a finite velocity in both the second and third layers, and was programmed for an IBM 704 computer. Bottom loss computations were then performed using typical sediment velocities, densities and absorptions for comparison with bottom loss measurements. In performing these calculations, the first layer was always assumed to be water of

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[22]See Footnotes 18 and 19.

[23]See Footnote 1.

[24]See Footnote 14.

[25]Schulkin, M., Thorp, W., Study D-"Report on the Status of Project AMOS (Acoustical, Meteoroligcal and Oceanographic Survey)(1 Jan 53-31 Dec 1954)", USL Report No. 255 (March 1955) (Confidential).

[26]Cole, B. F., Bell, T. G., "Three Layer Fluid Model for Ocean Bottom Reflectivity", USL Technical Memorandum No. 905-55-62 (December 1962)

[27]See Footnote 26.

a constant velocity and density, the second layer was always assumed to be a fairly transparent, low-velocity sediment (clay or silt) of varying thickness, and the third layer was always assumed to be a fairly firm, high velocity sediment (silt or sand). Such assumptions concerning the penetration and reflectivity of various types of sediment are believed justifiable according to the experimental observations previously discussed.

Theoretical losses computed for a silt-over-sand bottom are compared with Mackenzie's 1 kc bottom loss measurements[28] in Figure 1. Theoretical curves were computed for various layer thicknesses, and the thickness providing the best curve fit to the data was chosen. This comparison indicates that the three-layer fluid model is able to predict the low scatter of losses for grazing angles less than twenty degrees, and yet account for the high scatter of losses for grazing angles greater than twenty degrees.

The good agreement obtained between measured and theoretical bottom loss curves has prompted the presentation of theoretical curves for various frequencies, bottom types, and layer thicknesses (see Figures 2-24). The theoretical curves presented in Figures 2-8 were computed for a frequency study using the same sediment constants, and varying only the absorption coefficients in the second and third layers to correspond to the variation in frequency.[29] The effect of layer depth on the theoretical bottom loss for various frequencies and various sediment constants is shown by Figures 9-24. These figures also provide some indication of the variation in bottom loss with changing sediment constants at a given frequency.

Although only one example of measured versus theoretical bottom loss is included in this paper, extensive comparisons of classified experimental[30, 31] and theoretical bottom losses, omitted for purposes of classification[32], have shown fairly good agreement, and indicate that this theory will provide a good approximation of ocean bottom reflectivity.

BERNARD F. COLE
Bernard F. Cole

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[28]See Footnote 14.

[29]Various relations between the attenuaion and the frequency were assumed, however, a linear variation in attenuation with variation in frequency provided the best fit to data ranging from 125 cps to 4 kc.

[30]See Footnote 14.

[31]BRASS II experimental data (unpublished).

[32]This report was originally written as a co-op report for Northeastern University, and hence necessarily unclassified.

APPENDIX A

DEFINITION OF TERMS

Density: $\frac{\text{wet weight of sediment}}{\text{total volume of sample}}$

Porosity: Ratio of volume of voids (sea water in this case) between the grains of a sediment sample to the total volume of the sediment.

Sand, Silt and Clay: Wentworth grade scale of particle diameter, i.e., sand 2 - 0.062 mm, silt 0.062 - 0.004 mm, and clay less than 0.004 mm.

Theoretical Reflection Loss in DB/Reflection = 20 $\log_{10}$ reflection amplitude coefficient.

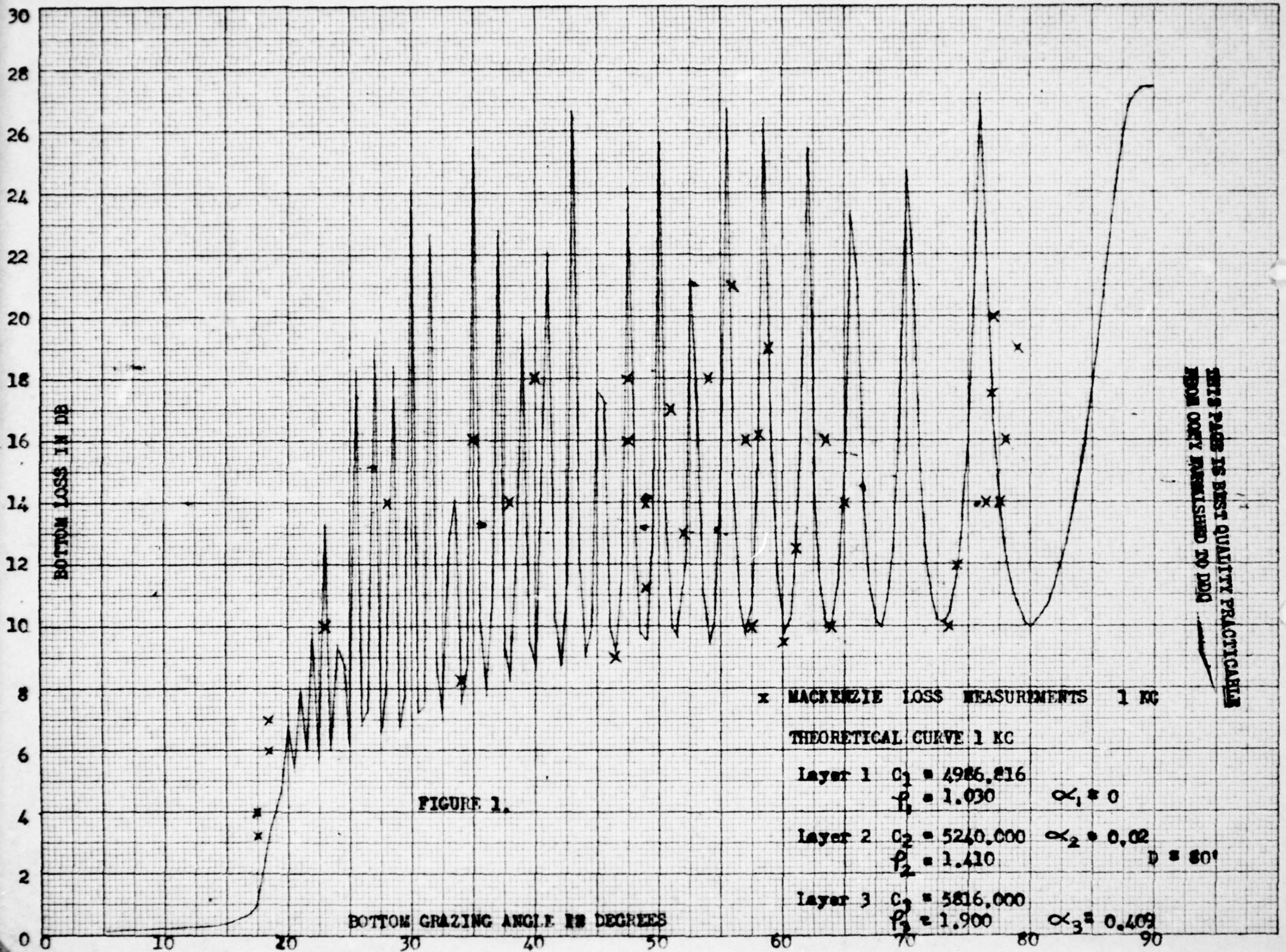

BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
0
10
20
30
40
50
60
70
80
90
x MACKENZIE LOSS MEASUREMENTS 1 KC
THEORETICAL CURVE 1 KC
Layer 1 C_1 = 4986.816 ρ_1 = 1.030 α_1 = 0
Layer 2 C_2 = 5240.000 ρ_2 = 1.410 α_2 = 0.02
Layer 3 C_3 = 5816.000 ρ_3 = 1.900 α_3 = 0.409
D = 80'

FIGURE 1.

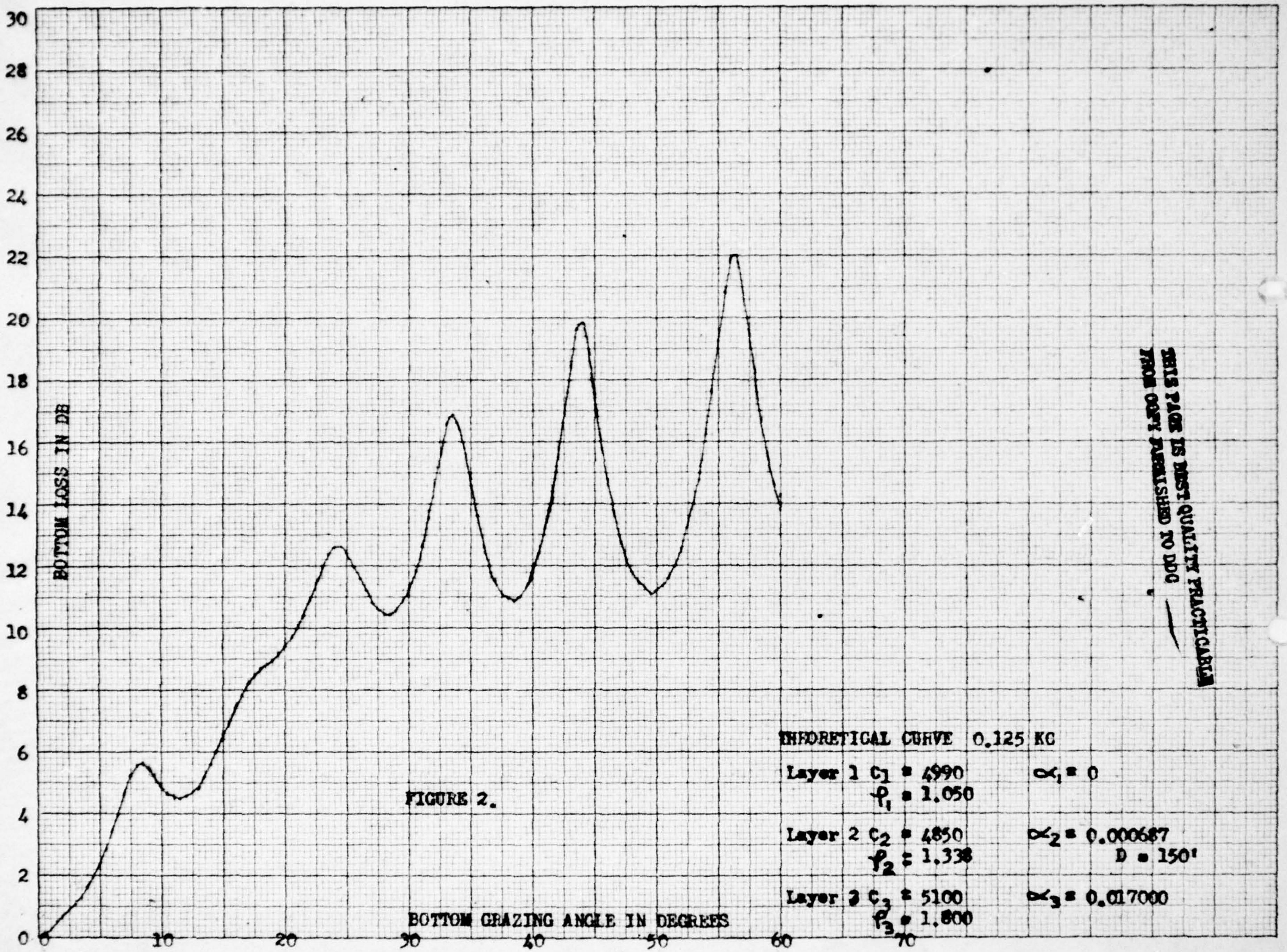

BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
THEORETICAL CURVE 0.125 KC
Layer 1 C_1 = 4990 ρ_1 = 1.050 α_1 = 0
Layer 2 C_2 = 4850 ρ_2 = 1.338 α_2 = 0.000687 D = 150'
Layer 3 C_3 = 5100 ρ_3 = 1.800 α_3 = 0.017000

FIGURE 2.

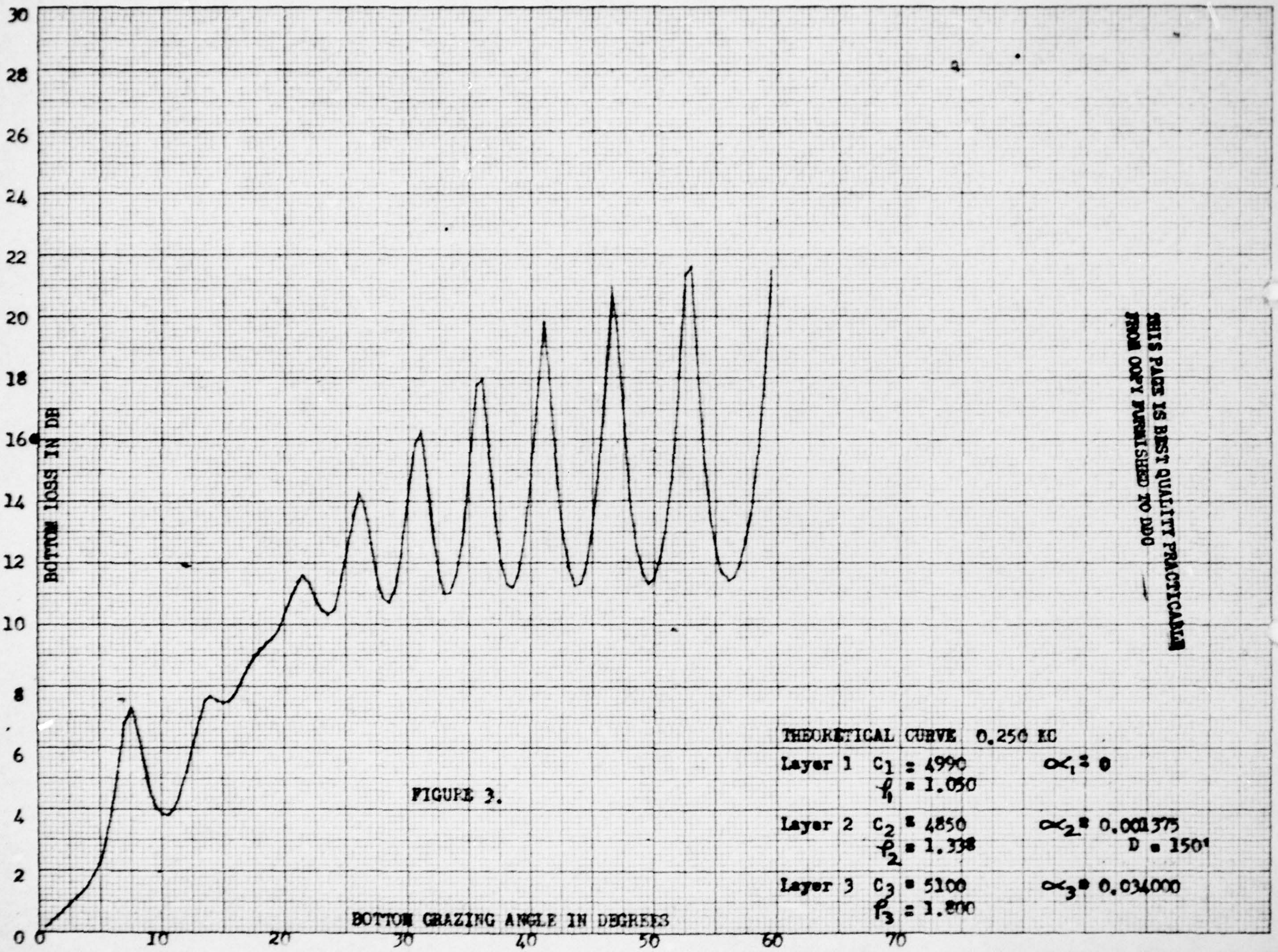

BOTTOM LOSS IN DB
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
BOTTOM GRAZING ANGLE IN DEGREES
0
10
20
30
40
50
60
70
THEORETICAL CURVE 0.250 KC
Layer 1 $C_1 = 4990$ $\rho_1 = 1.050$ $\alpha_1 = 0$
Layer 2 $C_2 = 4850$ $\rho_2 = 1.338$ $\alpha_2 = 0.001375$ D = 150'
Layer 3 $C_3 = 5100$ $\rho_3 = 1.800$ $\alpha_3 = 0.034000$

FIGURE 3.

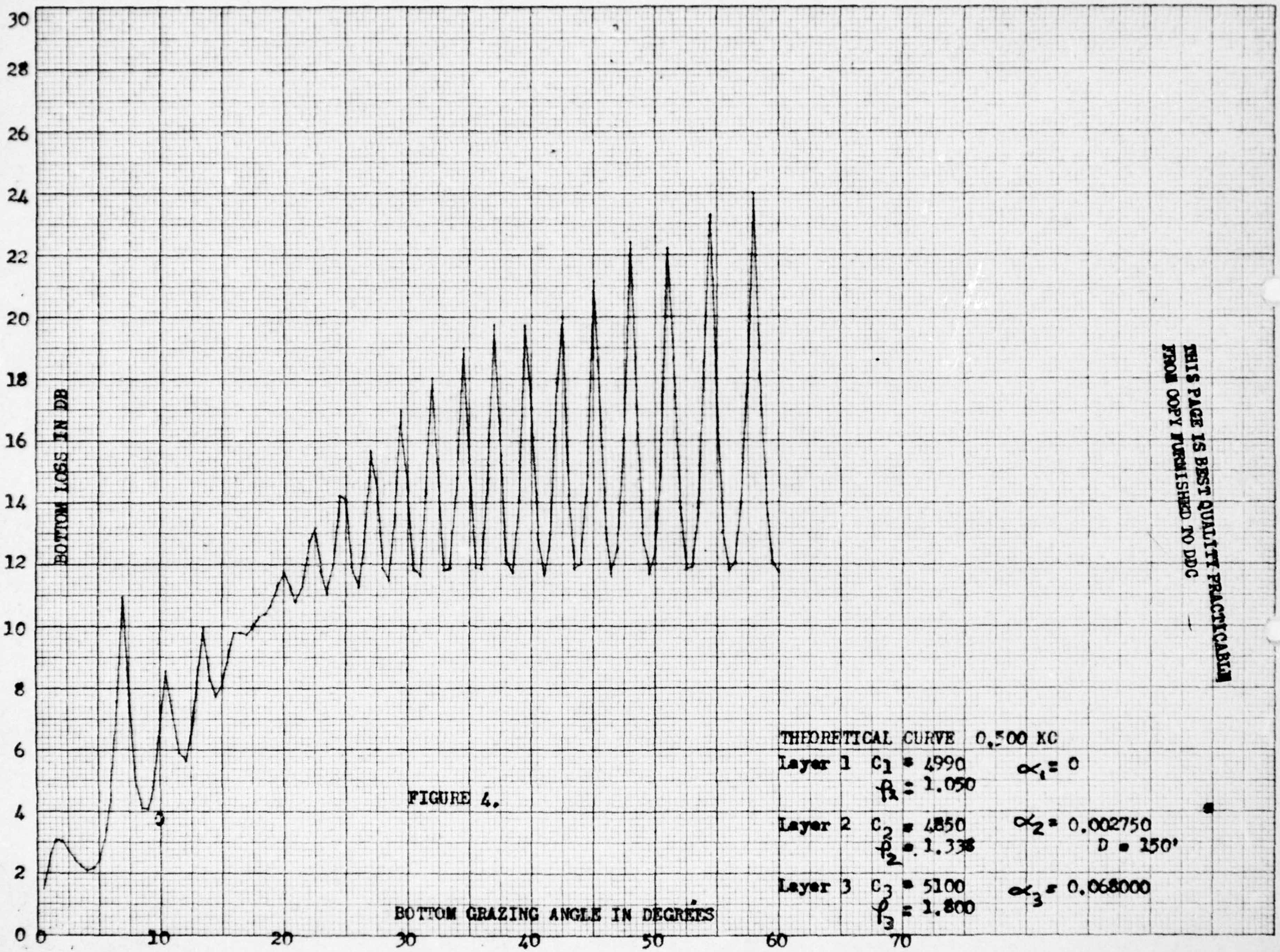

BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
0
10
20
30
40
50
60
70
THEORETICAL CURVE 0.500 KC
Layer 1 C_1 = 4990 ρ_1 = 1.050 α_1 = 0
Layer 2 C_2 = 4850 ρ_2 = 1.338 α_2 = 0.002750 D = 150'
Layer 3 C_3 = 5100 ρ_3 = 1.800 α_3 = 0.068000

FIGURE 4.

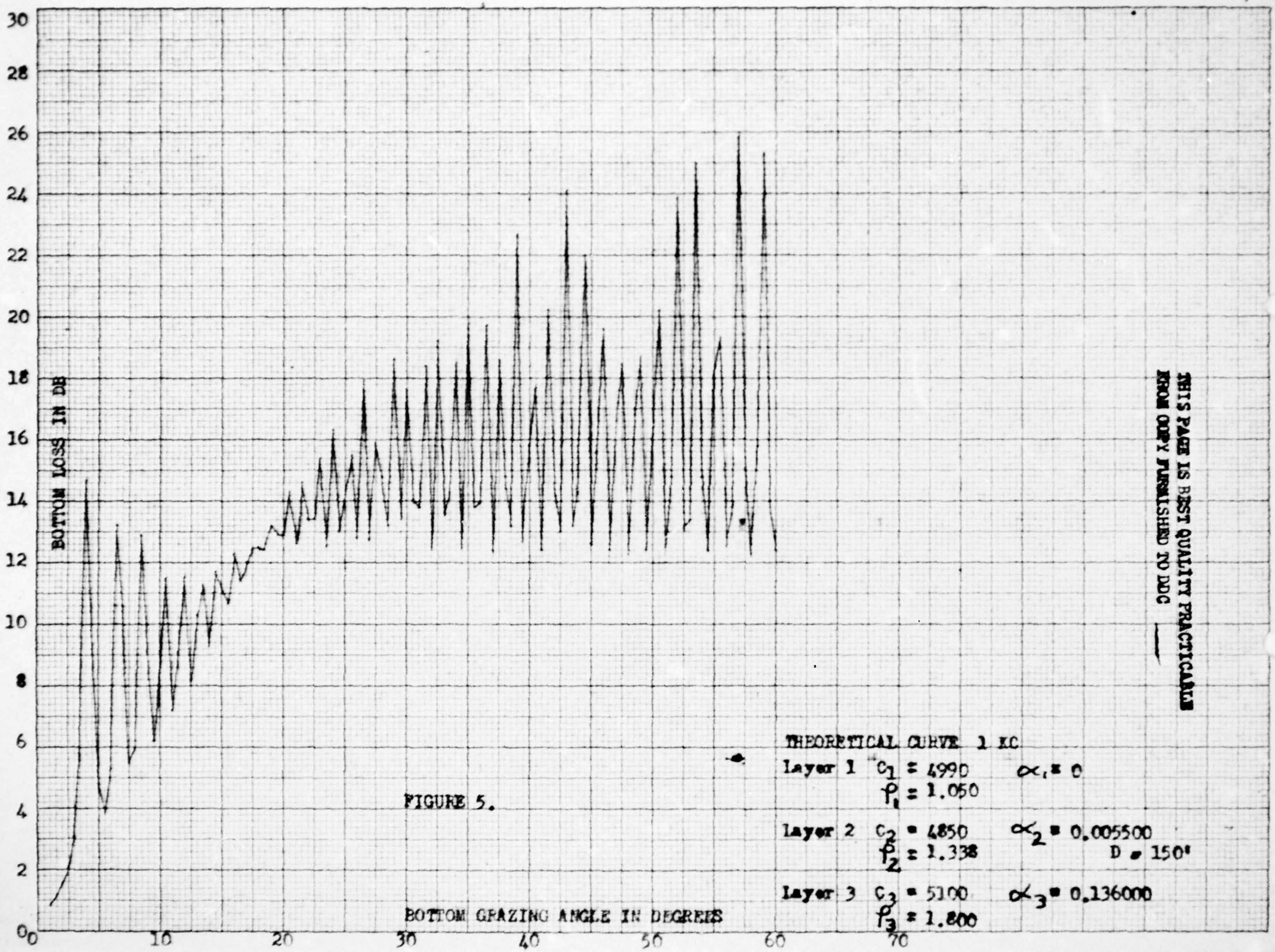
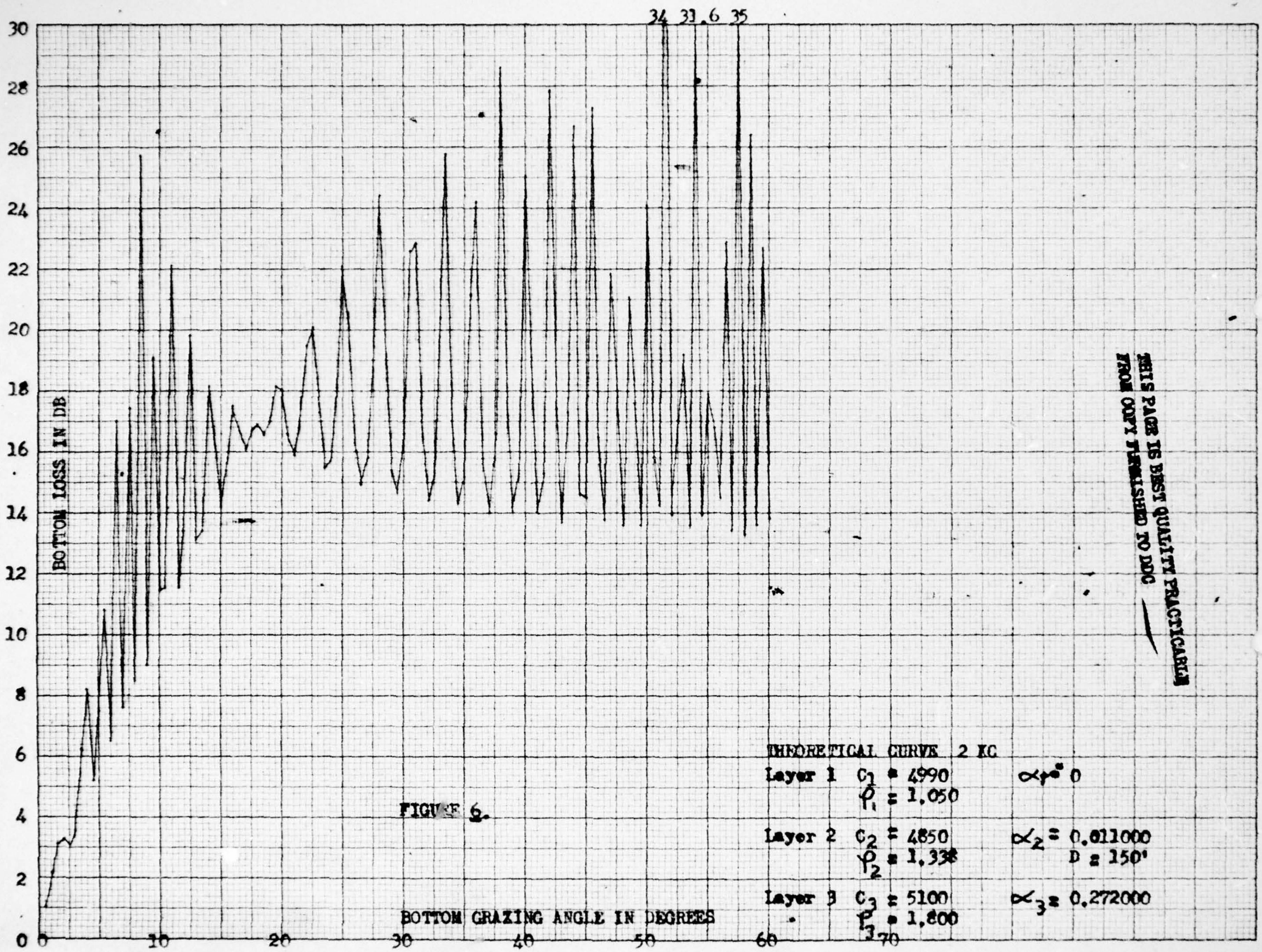

BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
0 10 20 30 40 50 60 70
THEORETICAL CURVE 1 KC
Layer 1 $C_1 = 4990$ $\alpha_1 = 0$
$\rho_1 = 1.050$
Layer 2 $C_2 = 4850$ $\alpha_2 = 0.005500$
$\rho_2 = 1.338$ D = 150'
Layer 3 $C_3 = 5100$ $\alpha_3 = 0.136000$
$\rho_3 = 1.800$

FIGURE 5.



34 33.6 35
BOTTOM LOSS IN DB
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
BOTTOM GRAZING ANGLE IN DEGREES
10
20
30
40
50
60
70
THEORETICAL CURVE 2 KC
Layer 1 C_1 = 4990 ρ_1 = 1,050 α_ρ = 0
Layer 2 C_2 = 4850 ρ_2 = 1,338 α_2 = 0.011000 D = 150'
Layer 3 C_3 = 5100 ρ_3 = 1,800 α_3 = 0.272000

FIGURE 6.

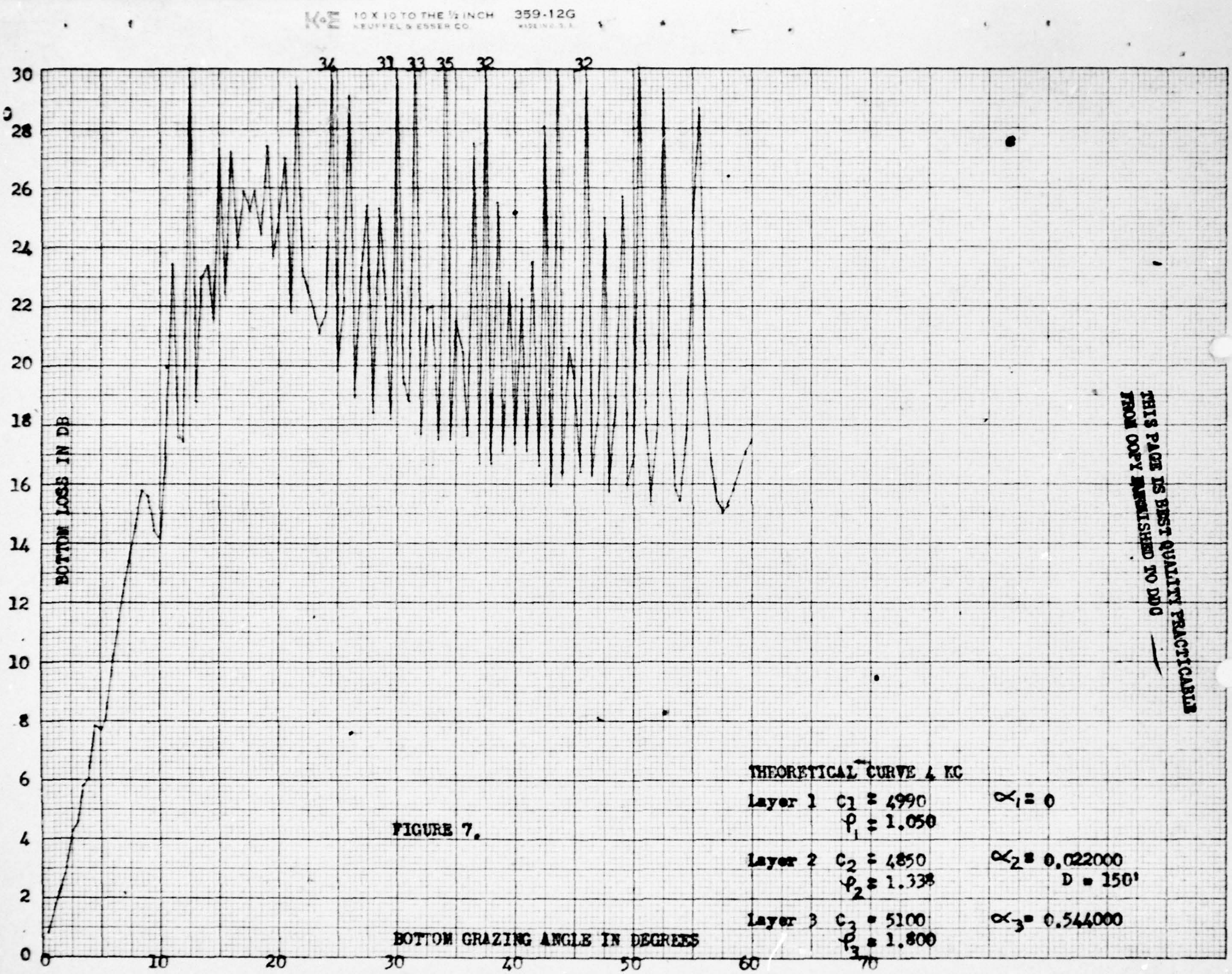

10 X 10 TO THE ½ INCH 359-12G
KEUFFEL & ESSER CO.
BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
34
31
33
35
32
32
THEORETICAL CURVE 4 KC
Layer 1 C1 = 4990 ρ1 = 1.050 α1 = 0
Layer 2 C2 = 4850 ρ2 = 1.338 α2 = 0.022000 D = 150'
Layer 3 C3 = 5100 ρ3 = 1.800 α3 = 0.544000

FIGURE 7.

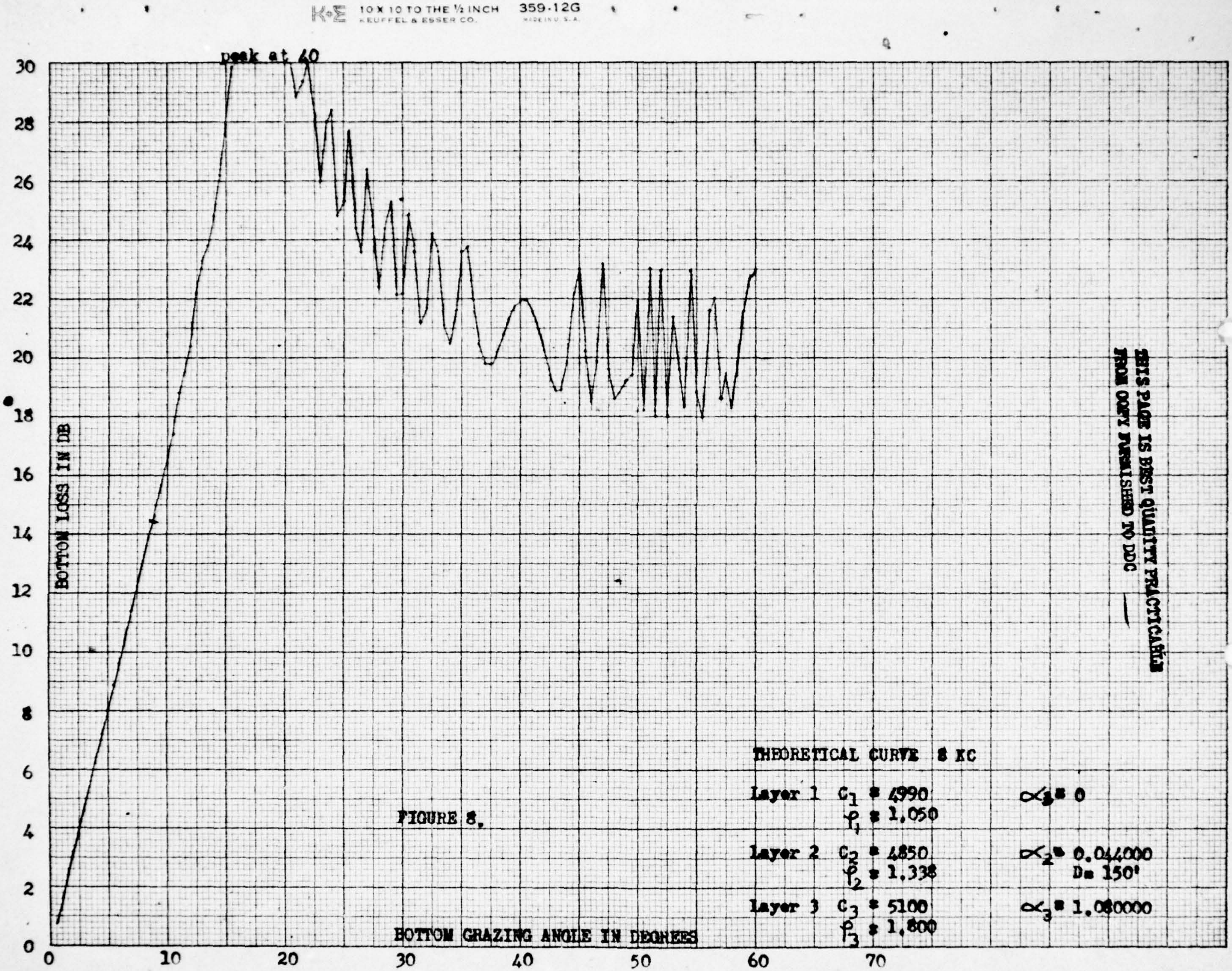

K+E 10 X 10 TO THE ½ INCH 359-12G
KEUFFEL & ESSER CO.
peak at 40
BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
10
20
30
40
50
60
70
THEORETICAL CURVE 8 KC
Layer 1 C_1 = 4990 ρ_1 = 1.050 α_1 = 0
Layer 2 C_2 = 4850 ρ_2 = 1.338 α_2 = 0.044000 D= 150'
Layer 3 C_3 = 5100 ρ_3 = 1.800 α_3 = 1.080000

FIGURE 8.

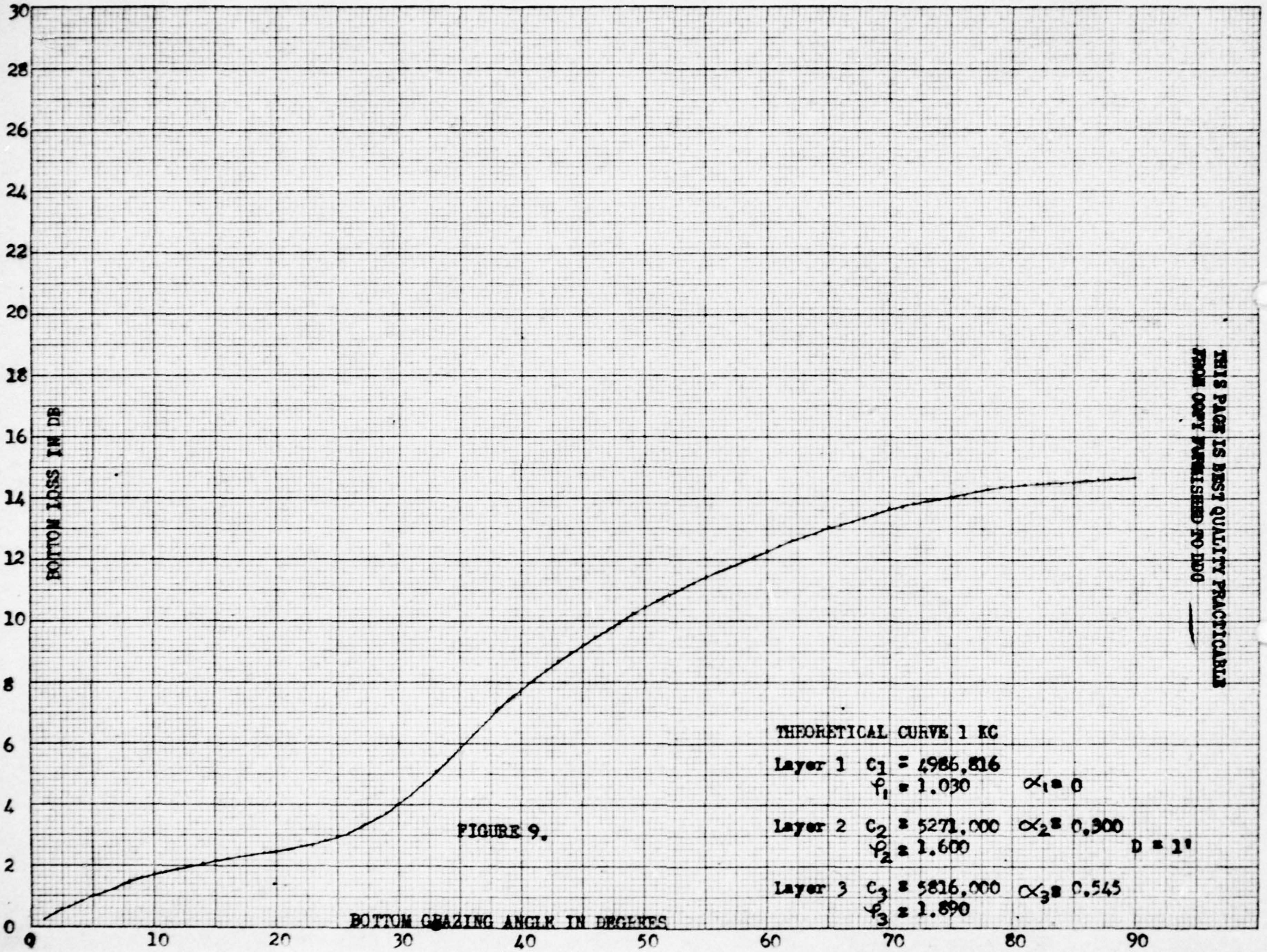

BOTTOM LOSS IN DB
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
BOTTOM GRAZING ANGLE IN DEGREES
0
10
20
30
40
50
60
70
80
90
THEORETICAL CURVE 1 KC
Layer 1 C_1 = 4986,816 ρ_1 = 1.030 α_1 = 0
Layer 2 C_2 = 5271.000 ρ_2 = 1.600 α_2 = 0.300
Layer 3 C_3 = 5816.000 ρ_3 = 1.890 α_3 = 0.545
D = 1'

FIGURE 9.

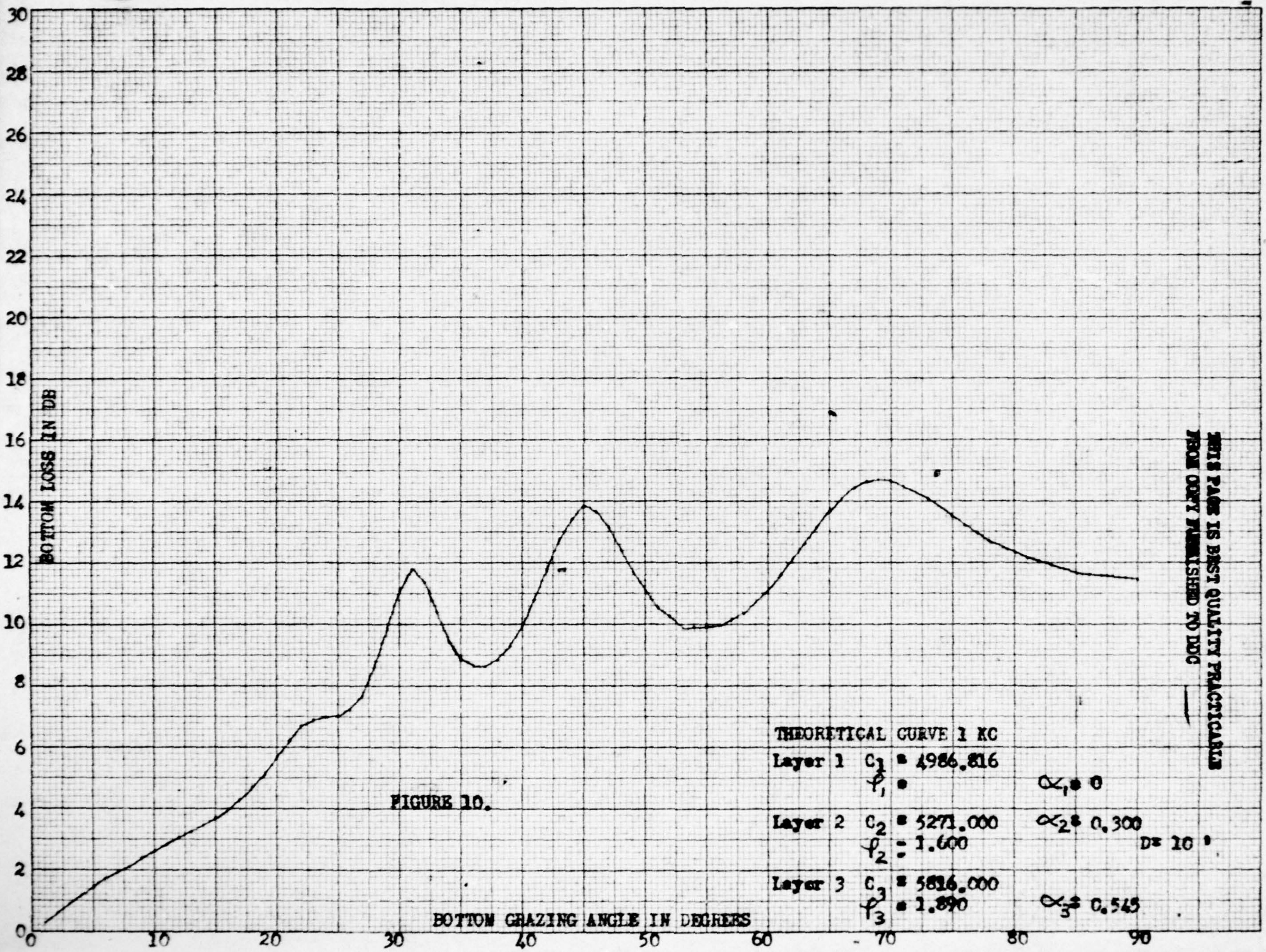

BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
0 10 20 30 40 50 60 70 80 90
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
THEORETICAL CURVE 1 KC
Layer 1 C1 = 4986.816
ρ1 =
α1 = 0
Layer 2 C2 = 5271.000
ρ2 = 1.600
α2 = 0.300
D = 10
Layer 3 C3 = 5816.000
ρ3 = 1.890
α3 = 0.545

FIGURE 10.

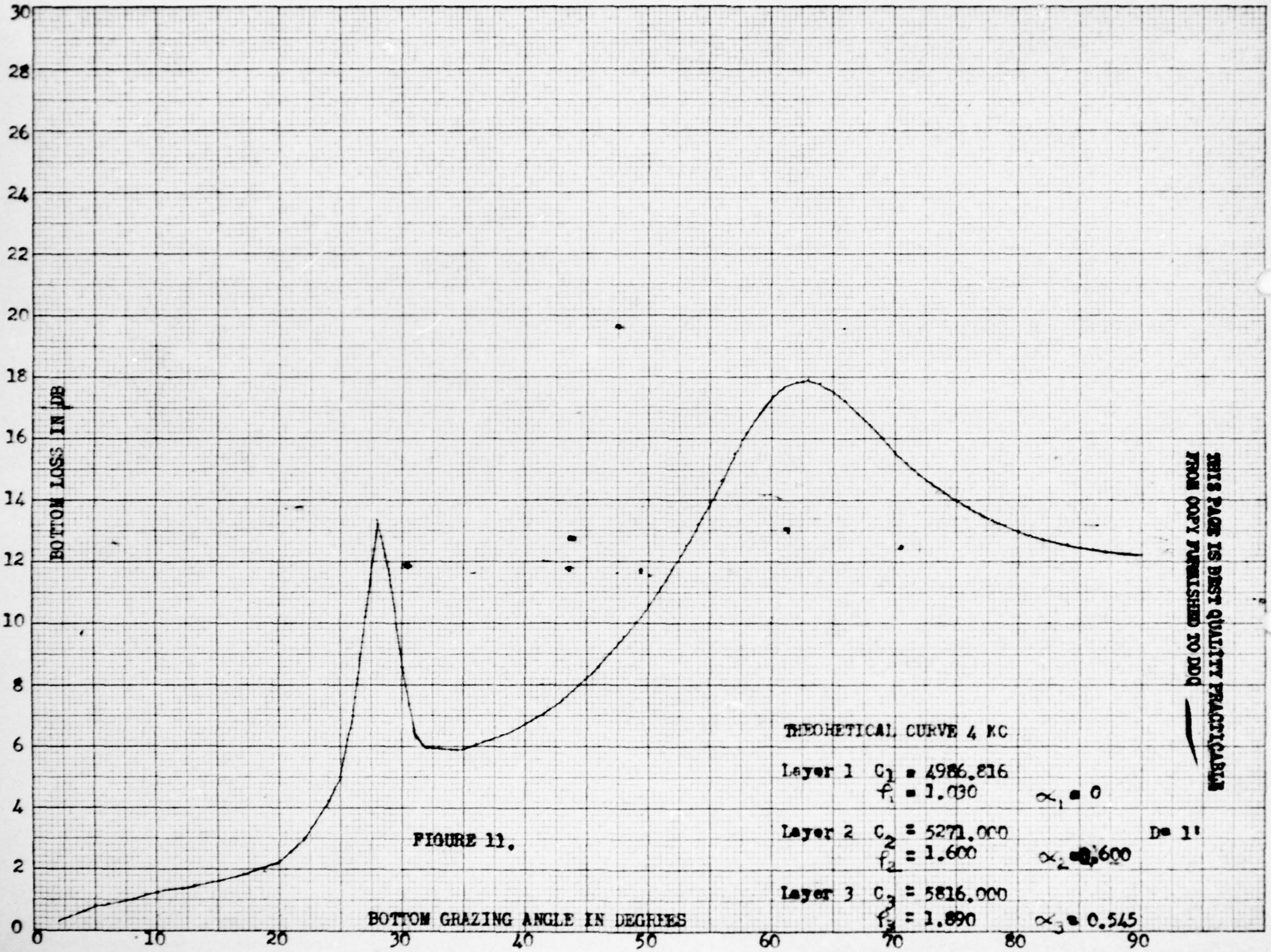
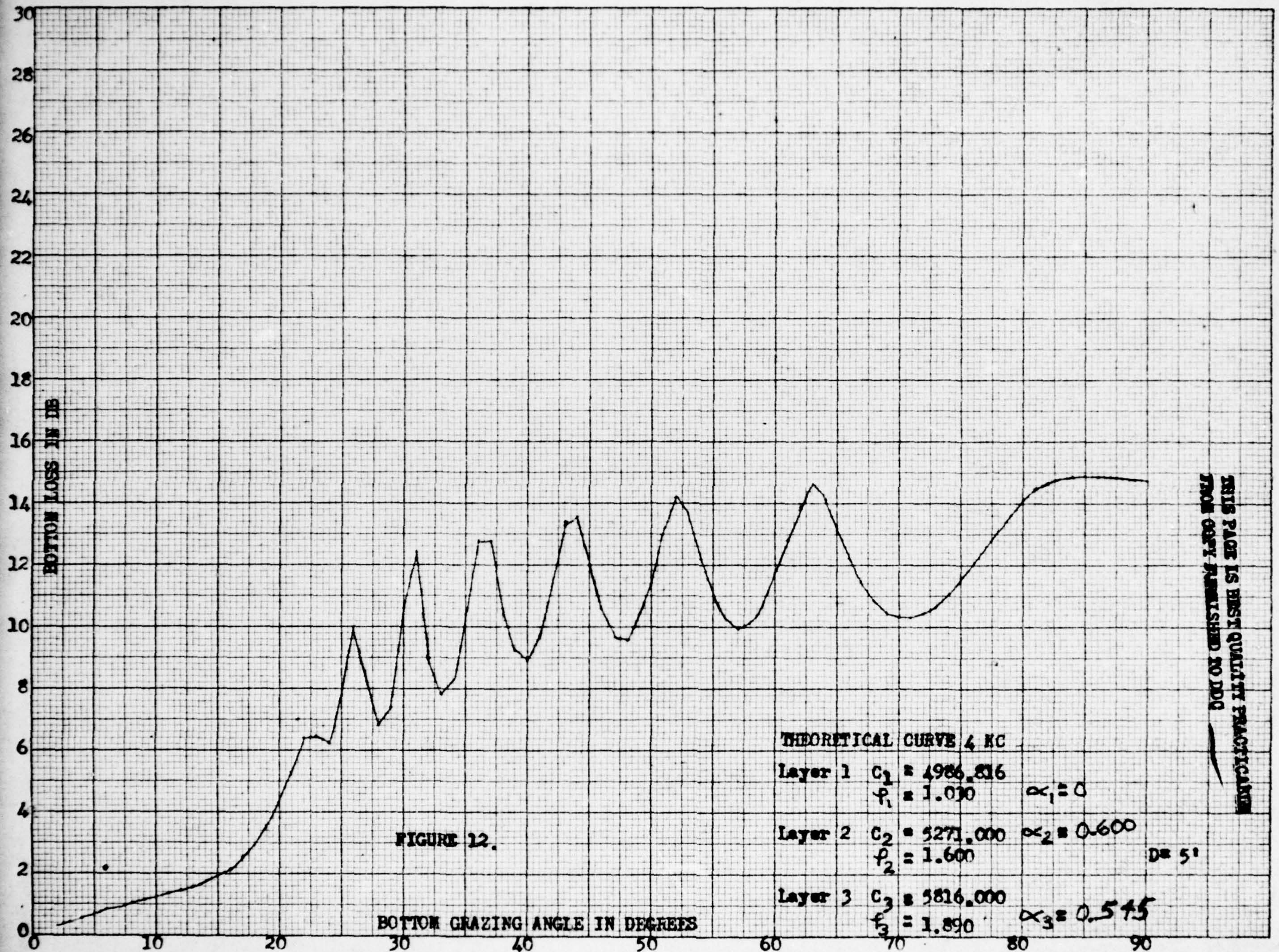

BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
10
20
30
40
50
60
70
80
90
THEORETICAL CURVE 4 KC
Layer 1 C_1 = 4986.816
ρ_1 = 1.030 α_1 = 0
Layer 2 C_2 = 5271.000 D= 1'
ρ_2 = 1.600 α_2 = 0.600
Layer 3 C_3 = 5816.000
ρ_3 = 1.890 α_3 = 0.545

FIGURE 11.



BOTTOM LOSS IN DB
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
0
10
20
30
40
50
60
70
80
90
BOTTOM GRAZING ANGLE IN DEGREES
THEORETICAL CURVE 4 KC
Layer 1 C_1 = 4986.816
ρ_1 = 1.030 α_1 = 0
Layer 2 C_2 = 5271.000 α_2 = 0.600
ρ_2 = 1.600
D = 5'
Layer 3 C_3 = 5816.000
ρ_3 = 1.890 α_3 = 0.545

FIGURE 12.

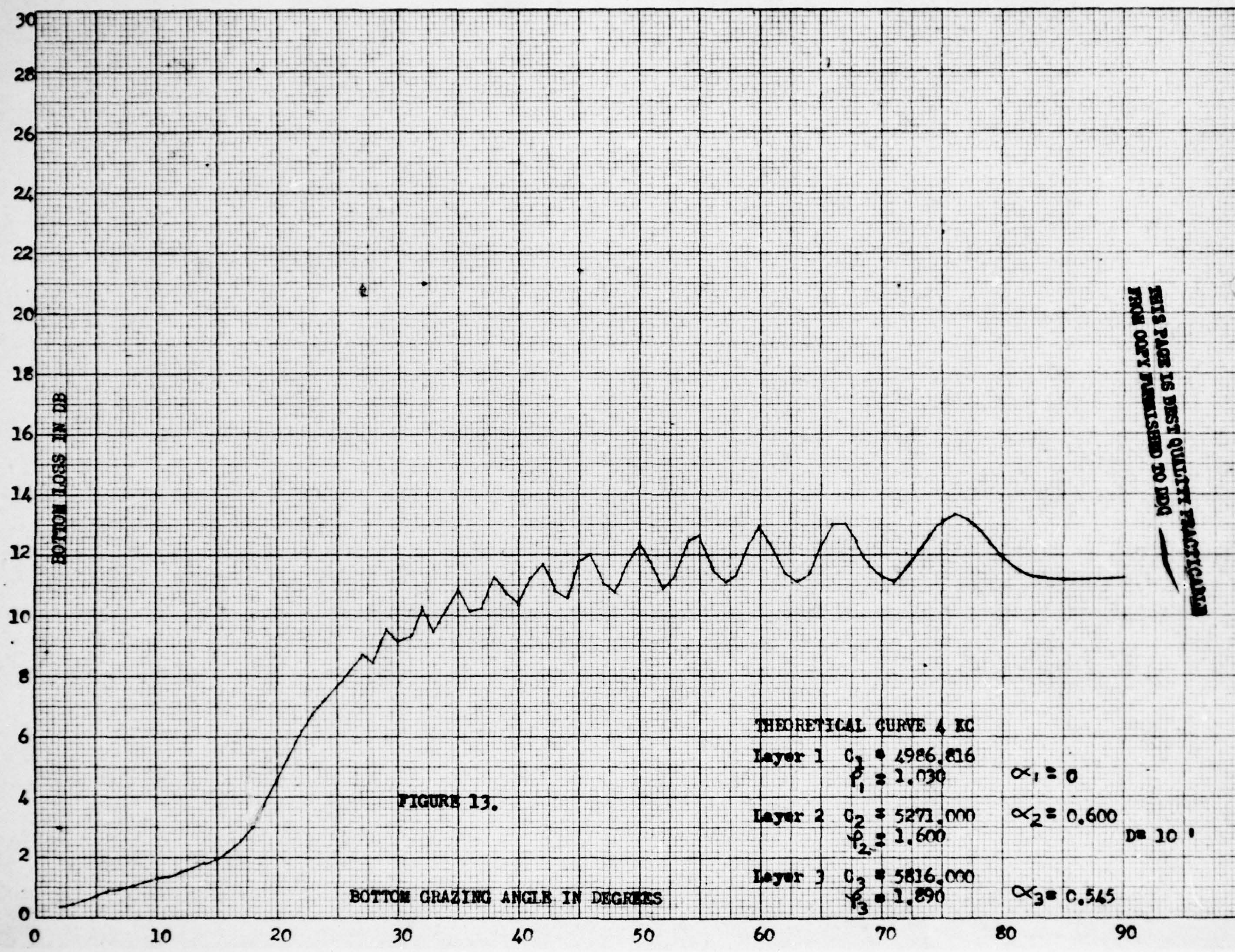

K+E 10 X 10 TO THE 1/2 INCH 359-12G
KEUFFEL & ESSER CO. MADE IN U.S.A.
BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
0
10
20
30
40
50
60
70
80
90
THEORETICAL CURVE 4 KC
Layer 1 C_1 = 4986.816
ρ_1 = 1.030
α_1 = 0
Layer 2 C_2 = 5271.000
ρ_2 = 1.600
α_2 = 0.600
D= 10'
Layer 3 C_3 = 5816.000
ρ_3 = 1.890
α_3 = 0.545

FIGURE 13.

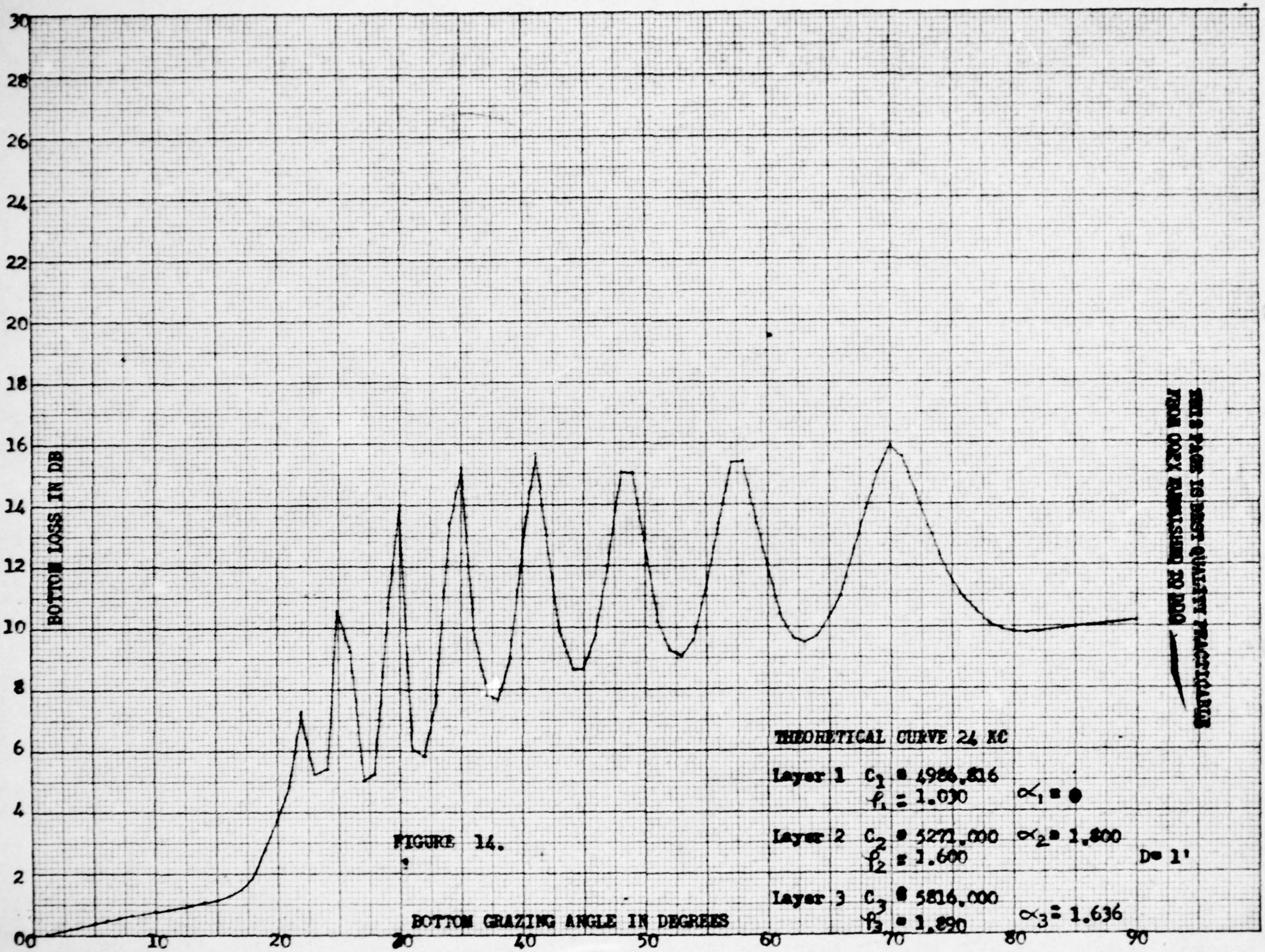

BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
10
20
30
40
50
60
70
80
90
THEORETICAL CURVE 24 KC
Layer 1 C_1 = 4986.816
ρ_1 = 1.030
α_1 = 0
Layer 2 C_2 = 5271.000
ρ_2 = 1.600
α_2 = 1.800
D = 1'
Layer 3 C_3 = 5816.000
ρ_3 = 1.890
α_3 = 1.636

FIGURE 14.

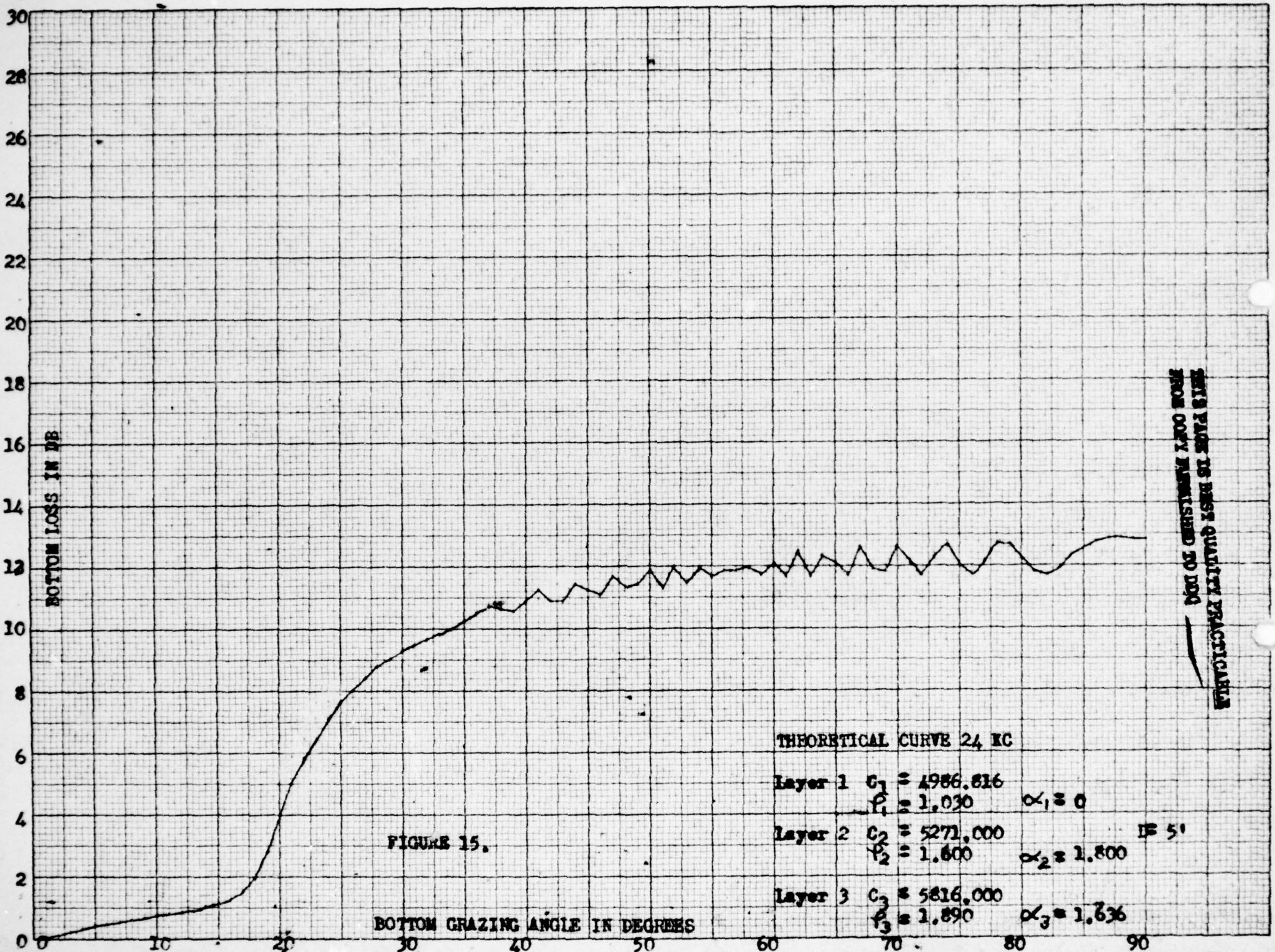

BOTTOM LOSS IN DB
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
BOTTOM GRAZING ANGLE IN DEGREES
0
10
20
30
40
50
60
70
80
90
THEORETICAL CURVE 24 KC
Layer 1 C_1 = 4986.816
ρ_1 = 1.030 α_1 = 0
Layer 2 C_2 = 5271.000
ρ_2 = 1.600 α_2 = 1.800
D = 5'
Layer 3 C_3 = 5816.000
ρ_3 = 1.890 α_3 = 1.636

FIGURE 15.

K+E 10 X 10 TO THE 1/2 INCH KEUFFEL & ESSER CO. 359-12G

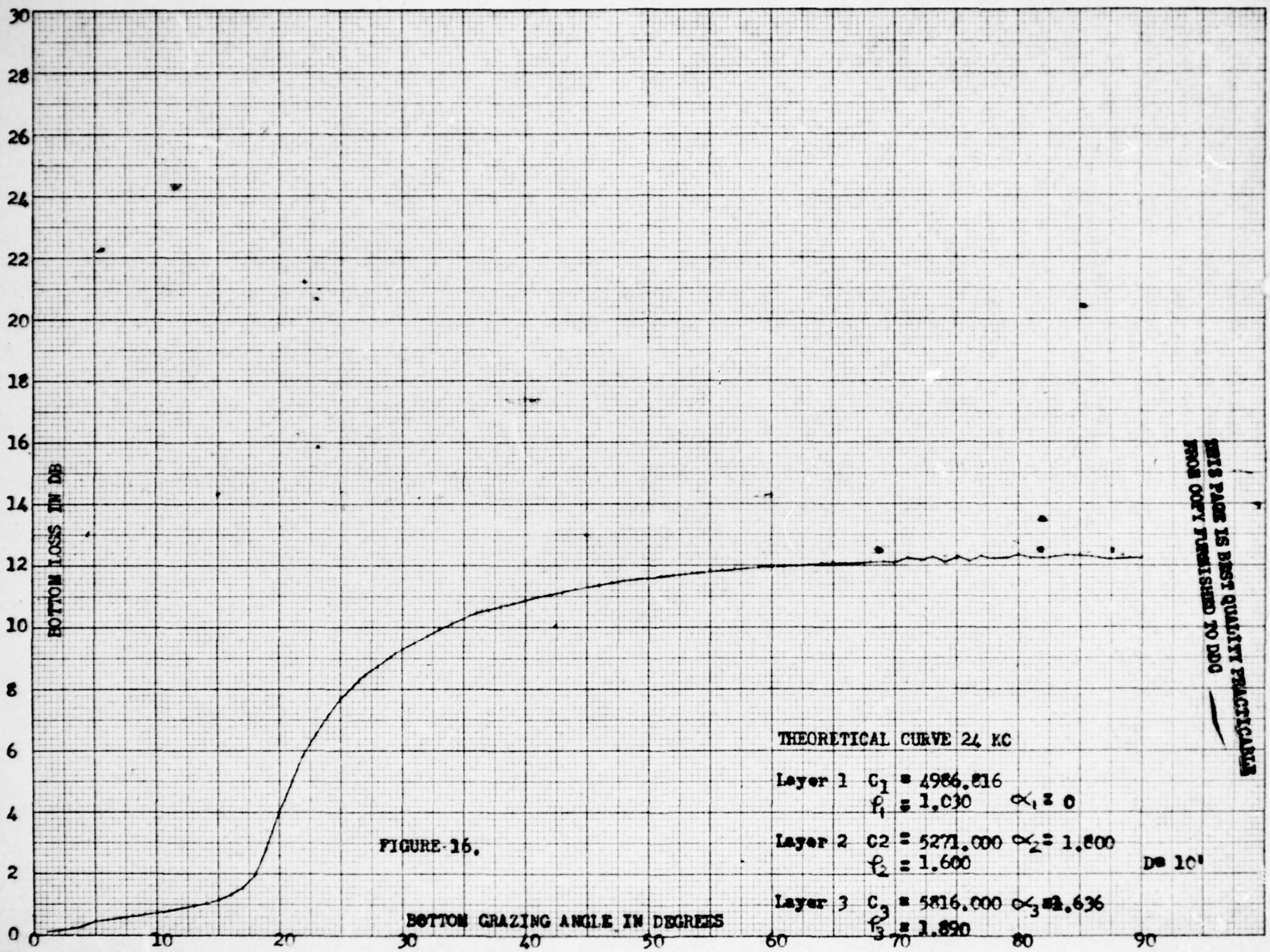


FIGURE 16.

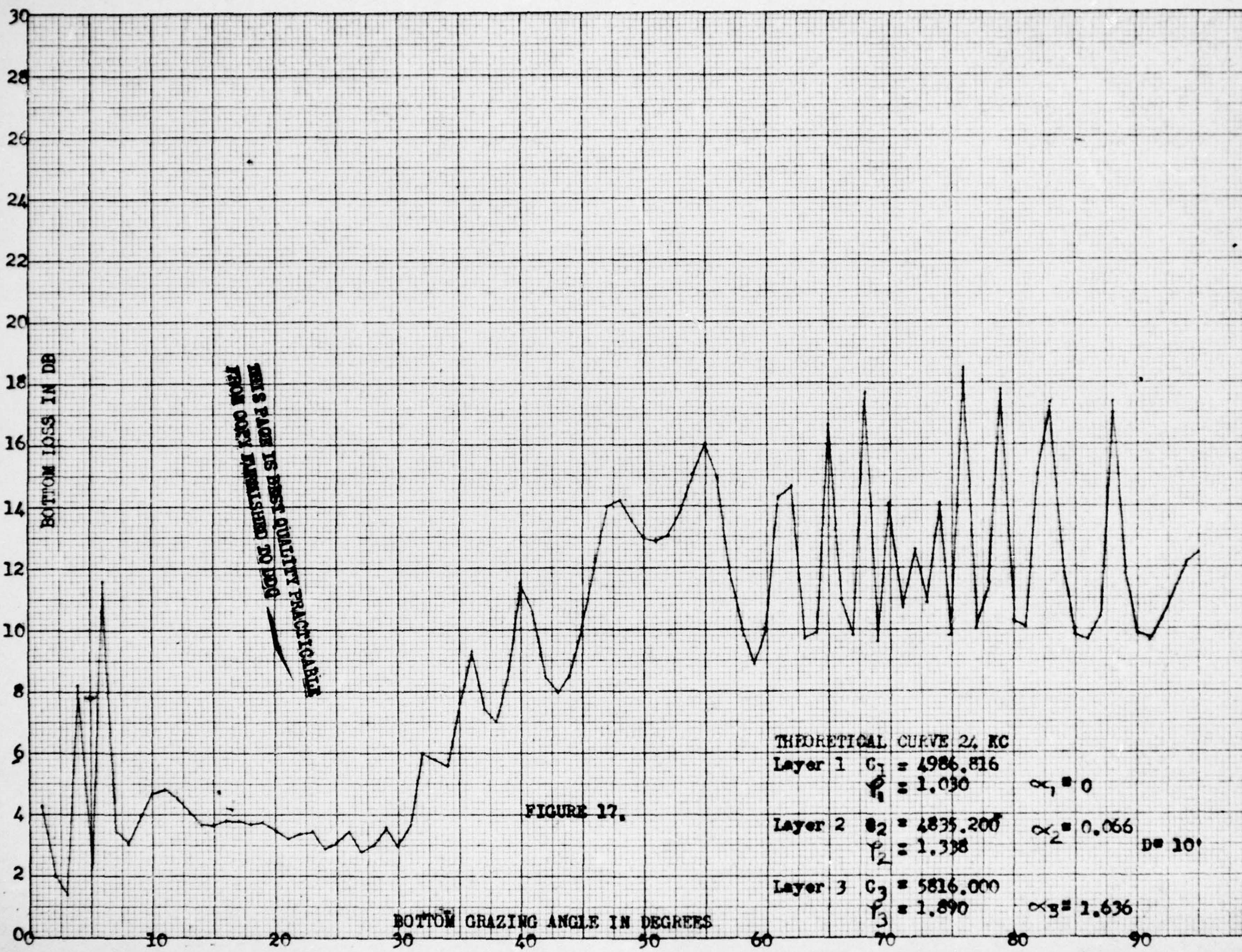

BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
0 10 20 30 40 50 60 70 80 90
THEORETICAL CURVE 24 KC
Layer 1 C_1 = 4986.816 ρ_1 = 1.030 α_1 = 0
Layer 2 C_2 = 4835.200 ρ_2 = 1.338 α_2 = 0.066
Layer 3 C_3 = 5816.000 ρ_3 = 1.890 α_3 = 1.636
D = 10'

FIGURE 17.

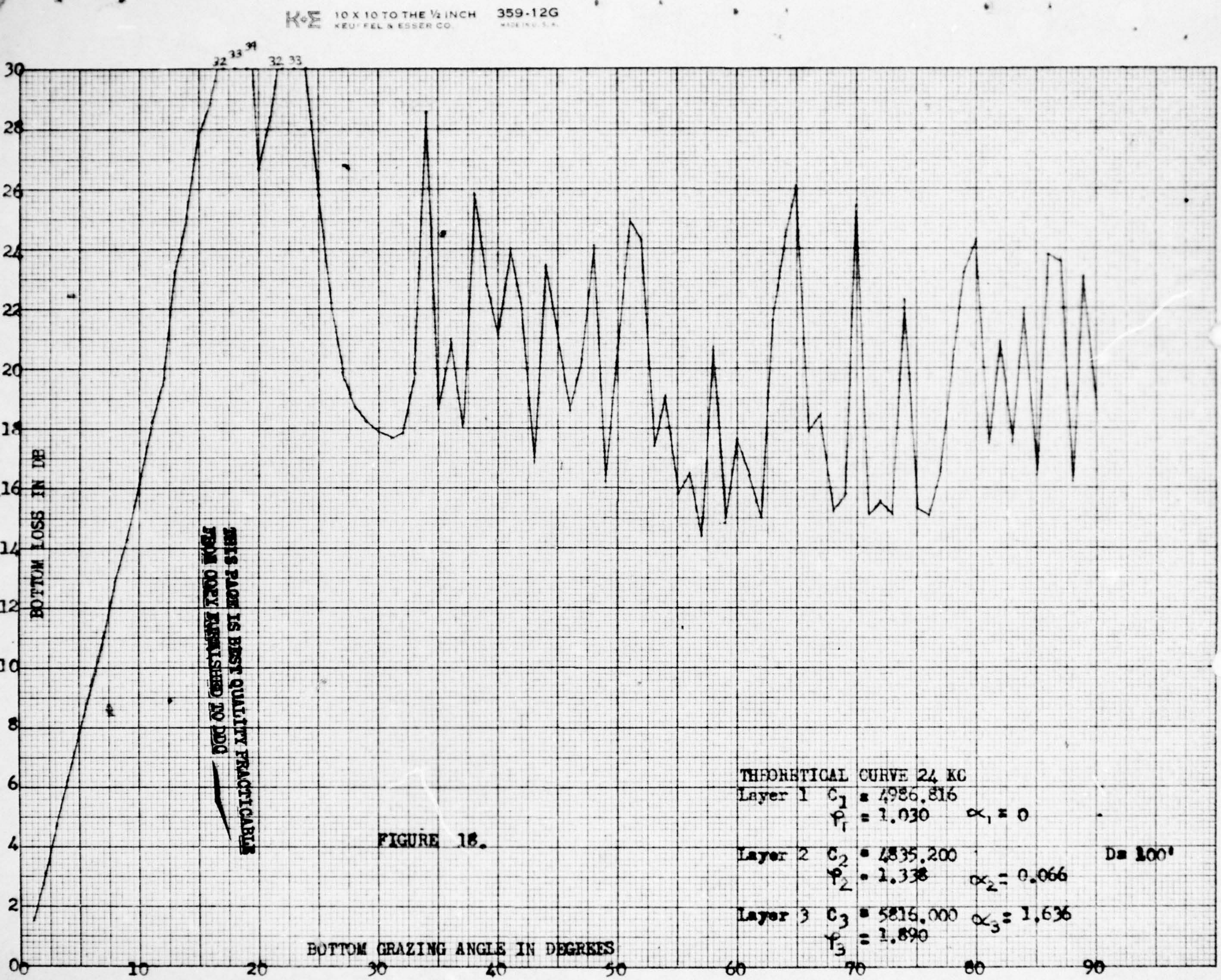

K+E 10 X 10 TO THE ½ INCH 359-12G
KEUFFEL & ESSER CO.
BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
32 33 34
32 33
THEORETICAL CURVE 24 KC
Layer 1 C_1 = 4986.816
ρ_1 = 1.030 α_1 = 0
Layer 2 C_2 = 4835.200
ρ_2 = 1.338 α_2 = 0.066
Layer 3 C_3 = 5816.000 α_3 = 1.636
ρ_3 = 1.890
D= 100'

FIGURE 18.

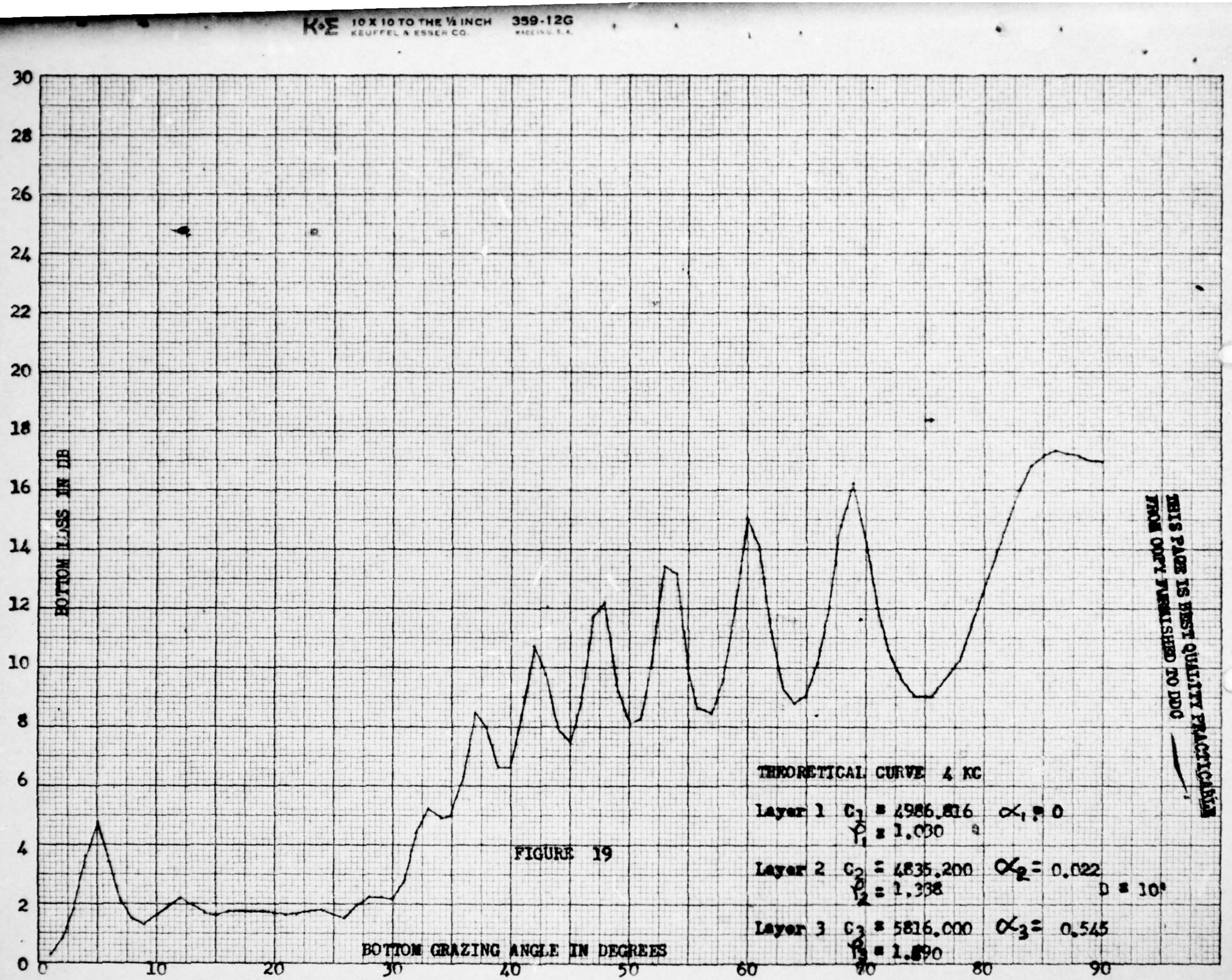

K+E 10 X 10 TO THE 1/2 INCH 359-12G
KEUFFEL & ESSER CO.
BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
10
20
30
40
50
60
70
80
90
THEORETICAL CURVE 4 KC
Layer 1 C_1 = 4986.816 α_1 = 0
ρ_1 = 1.030
Layer 2 C_2 = 4835.200 α_2 = 0.022
ρ_2 = 1.338
D = 10'
Layer 3 C_3 = 5816.000 α_3 = 0.545
ρ_3 = 1.890

FIGURE 19

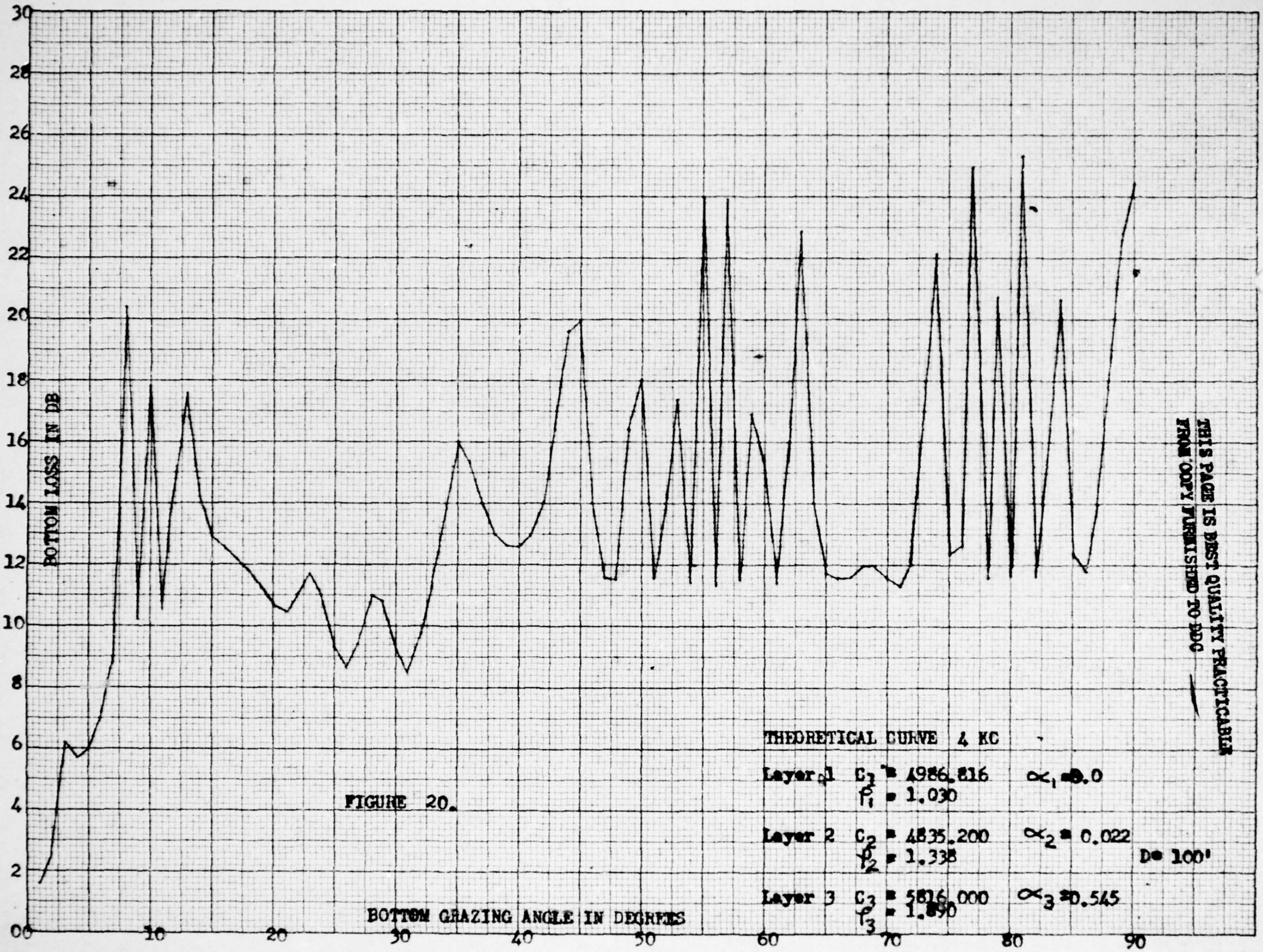

BOTTOM LOSS IN DB
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
BOTTOM GRAZING ANGLE IN DEGREES
10
20
30
40
50
60
70
80
90
THEORETICAL CURVE 4 KC
Layer 1 C_1 = 4986.816 α_1 = 0.0
ρ_1 = 1.030
Layer 2 C_2 = 4835.200 α_2 = 0.022
ρ_2 = 1.338
D = 100'
Layer 3 C_3 = 5816.000 α_3 = 0.545
ρ_3 = 1.890

FIGURE 20.

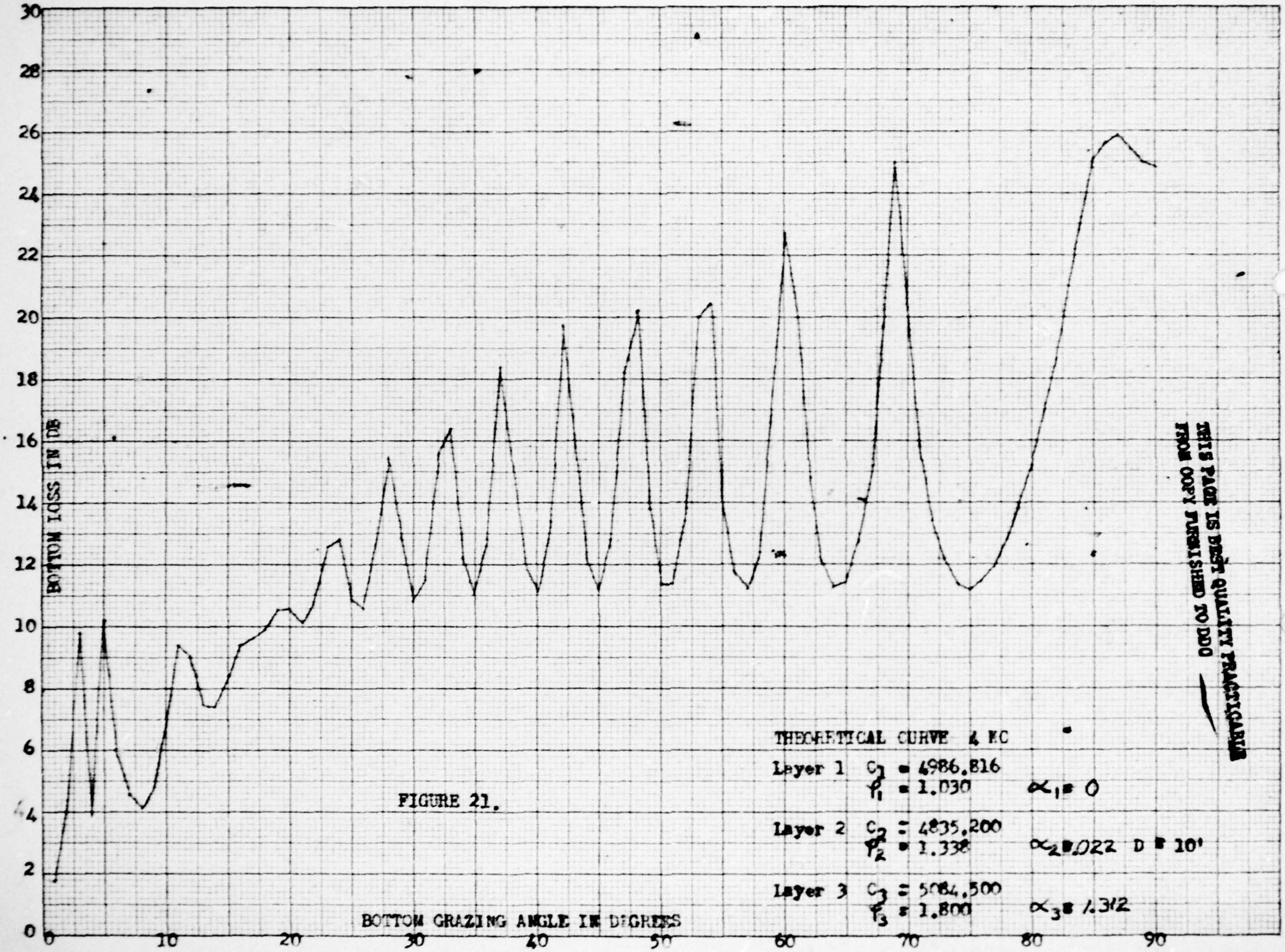

BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
THEORETICAL CURVE 4 KC
Layer 1 C_1 = 4986.816 ρ_1 = 1.030 α_1 = 0
Layer 2 C_2 = 4835.200 ρ_2 = 1.338 α_2 = .022 D = 10'
Layer 3 C_3 = 5084.500 ρ_3 = 1.800 α_3 = 1.312

FIGURE 21.

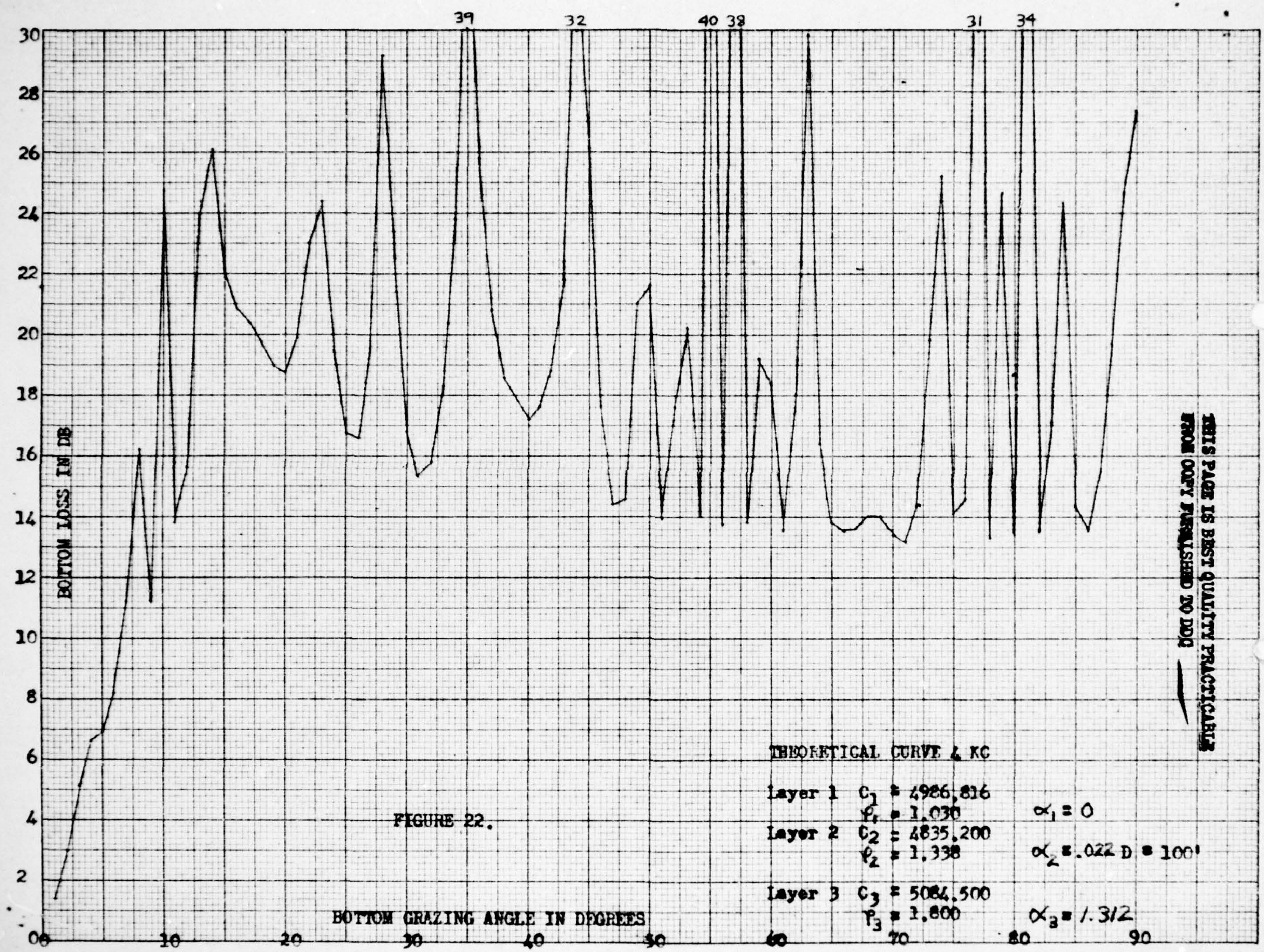

K+E 10 X 10 TO THE ½ INCH KEUFFEL & ESSER CO. 359-12G MADE IN U.S.A.
BOTTOM LOSS IN DB
BOTTOM GRAZING ANGLE IN DEGREES
39
32
40 38
31
34
THEORETICAL CURVE & KC
Layer 1 C_1 = 4986.816
ρ_1 = 1.030
$\alpha_1 = 0$
Layer 2 C_2 = 4835.200
ρ_2 = 1.338
α_2 = .022 D = 100'
Layer 3 C_3 = 5084.500
ρ_3 = 1.800
α_3 = 1.312

FIGURE 22.

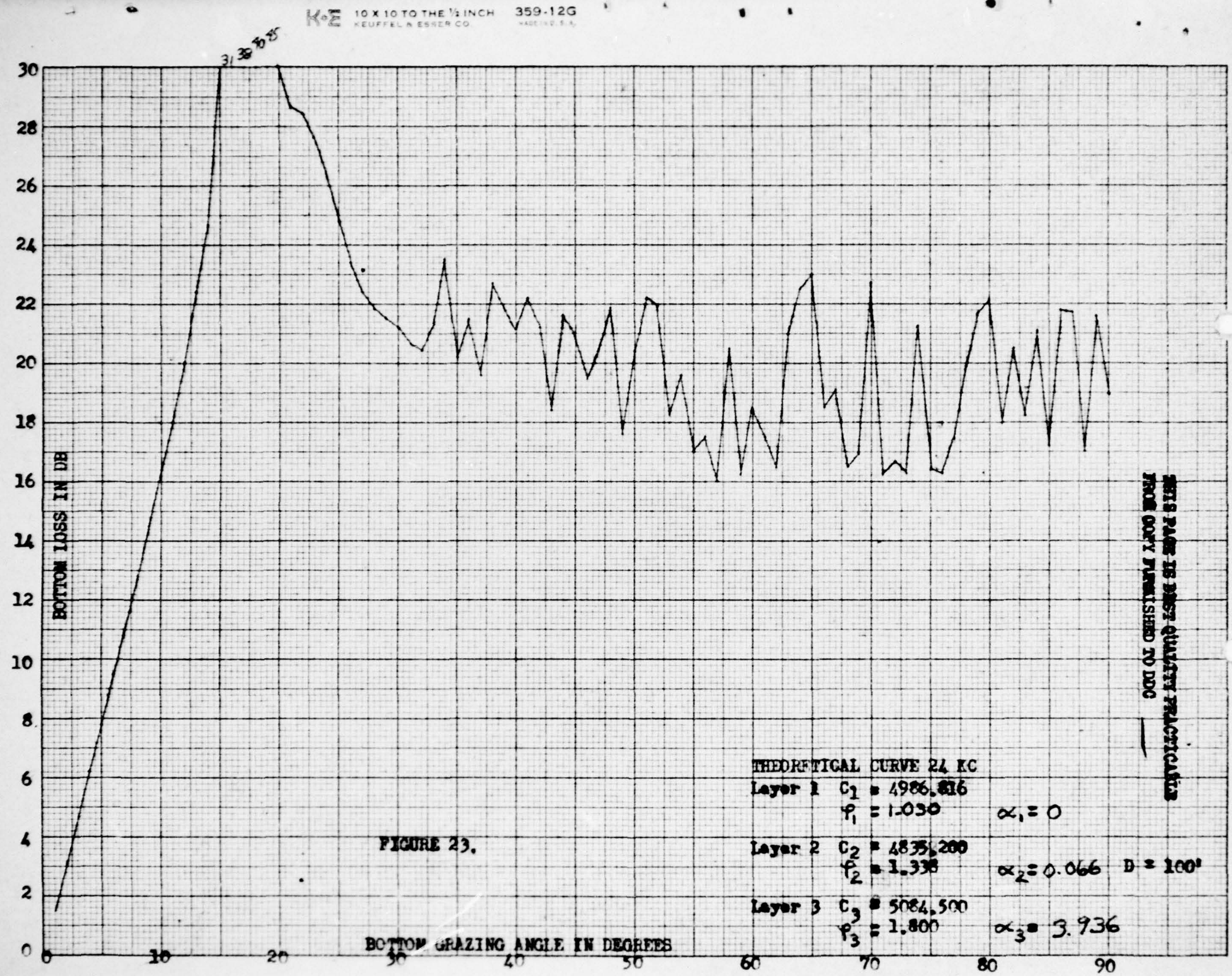

K+E 10 X 10 TO THE ½ INCH 359-12G
KEUFFEL & ESSER CO.
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
BOTTOM LOSS IN DB
0
10
20
30
40
50
60
70
80
90
BOTTOM GRAZING ANGLE IN DEGREES
31 38 40 45
THEORETICAL CURVE 24 KC
Layer 1 C1 = 4986.816
ρ1 = 1.030 α1 = 0
Layer 2 C2 = 4835.200
ρ2 = 1.338 α2 = 0.066 D = 100'
Layer 3 C3 = 5084.500
ρ3 = 1.800 α3 = 3.936

FIGURE 23.

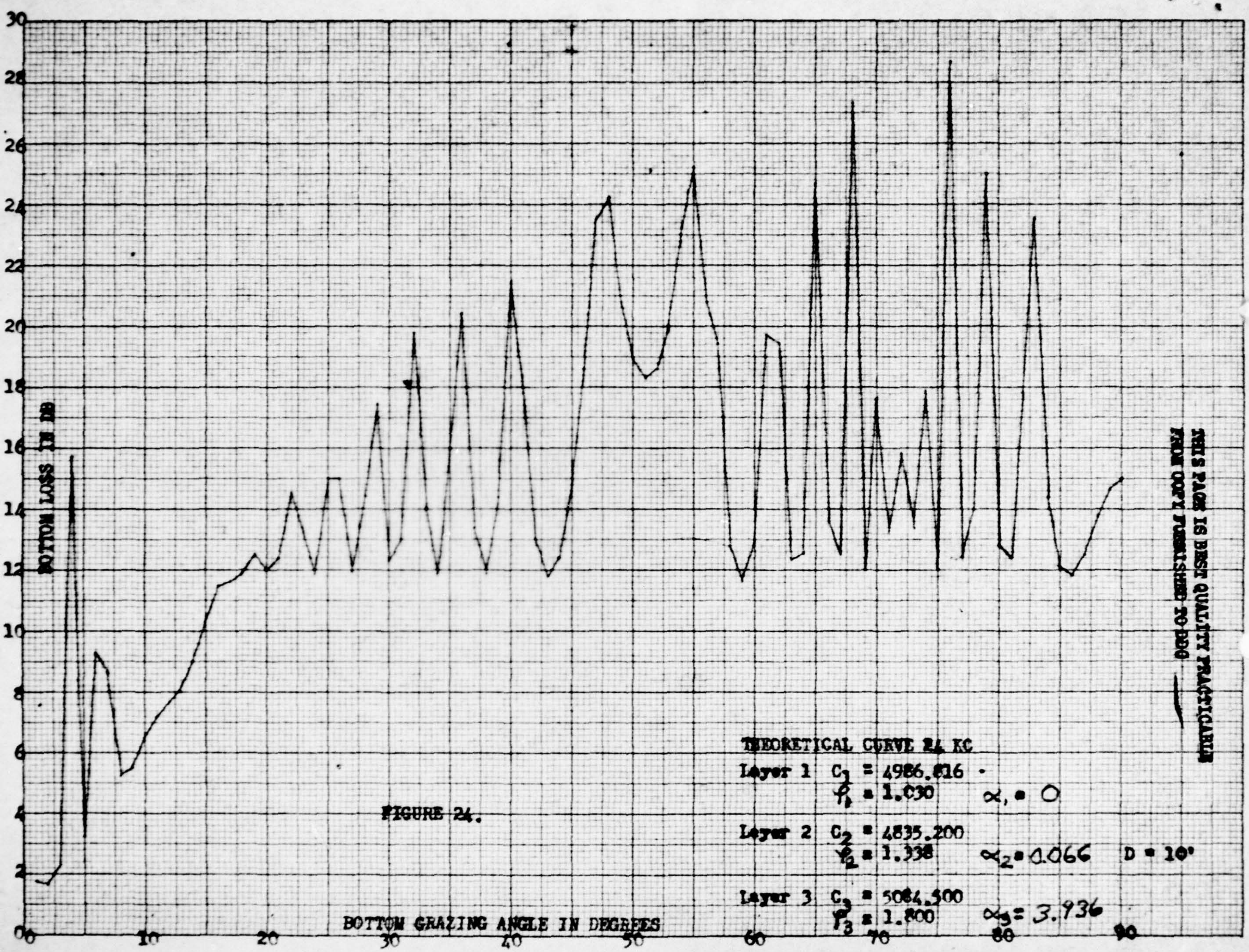

BOTTOM LOSS IN DB
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
BOTTOM GRAZING ANGLE IN DEGREES
0
10
20
30
40
50
60
70
80
90
THEORETICAL CURVE 24 KC
Layer 1 C_1 = 4986.816 ρ_1 = 1.030 α_1 = 0
Layer 2 C_2 = 4835.200 ρ_2 = 1.338 α_2 = 0.066 D = 10'
Layer 3 C_3 = 5084.500 ρ_3 = 1.800 α_3 = 3.936

FIGURE 24.